52.5|100

KOSOR

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: MATEO BOBAČEK **Broj indeksa:** 17-2-0113-2011

Vrijeme: od_____ **do**_____ **♣A** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Riješi sljedeći sustav jednadžbi:

$$\begin{aligned} x + 2y - 3z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z^2 + \frac{(1-i)^3}{i^{25}} = 0$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = \frac{3x+1}{(x-1)^2}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = \frac{1}{\ln(x-3)}$$

b) Odredi domenu funkcije:

$$f(x) = \sqrt{\ln(\frac{2x-1}{2x+7})}$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \frac{x^2 - 2x + 5}{x - 1}$$

② $z^2 + \frac{(1-i)^3}{i^{25}} =$

$z^2 = -\frac{(1-i)^3}{i} \cdot \frac{i}{i}$

$z^2 = -\frac{i(1-i)^3}{i^2}$

$z^2 = -\frac{i(1-i)^3}{-1}$

$z^2 = i(1-i)^3$

6

③ $f(x) = \frac{3x+1}{(x-1)^2} = \frac{3x+1}{x^2-2x-1}$

Domena

$(x-1)^2 \neq 0$ /$\sqrt{\ }$

$x-1 \neq$

$x \neq 1$ $\quad D(f) = \mathbb{R} \setminus \{1\}$

VA. $\lim_{x \to 1^-} \frac{3x+1}{(x-1)^2} = \frac{4}{0^-} = +\infty$

$\lim_{x \to 1^+} \frac{3x+1}{(x-1)^2} = \frac{4}{0^+} = +\infty$

VA... $x = 1$

15

HA. $\lim_{x \to \infty} \frac{3x+1}{(x-1)^2} = \lim_{x \to \infty} \frac{3x+1 /:x^2}{x^2-2x+1 /:x^2} = \lim_{x \to \infty} \frac{\frac{3}{x} + \frac{1}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = \frac{0}{1} = 0$ $\quad$ HA... $y = 0$

K.A. nema

④ a) $f(x) = \frac{1}{\ln(x-3)} = (\ln(x-3))^{-1}$

$f'(x) = -1 \cdot (\ln(x-3))^{-2} \cdot \frac{1}{x-3} \cdot 1$

$= -(\ln(x-3))^{-2} \cdot \frac{1}{x-3}$ //

12.5

b) $f(x) = \sqrt{\ln\left(\frac{2x-1}{2x+7}\right)}$

$\ln\left(\frac{2x-1}{2x+7}\right) \geq 0 / \cdot \exp^e$

$\frac{2x-1}{2x+7} \geq 1$

$\frac{2x-1}{2x+7} - 1 \geq 0$

$\frac{2x-1-2x-7}{2x+7} \geq 0$

$\frac{-8}{2x+7} \geq 0$

$2x+7 \geq 0$

$2x \geq -7$

$\frac{2x-1}{2x+7} > 0$

$x = \frac{1}{2}$ $\quad x = -\frac{7}{2}$

$2x+7 \neq 0$

$2x \neq -7$

$x \neq -\frac{7}{2}$

	$-\infty$		$\frac{1}{2}$		$+\infty$
$2x-1$		−	0	+	

$D(f) = \langle \frac{1}{2}, +\infty \rangle \setminus \{-\frac{7}{2}\}$

MATEO BOBAČEK

1.

♣2 A

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 2 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 1 & -1 & 3 & -5 & | & 6 \end{bmatrix} \cdot(-1) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 1 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(-3) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 1 & | & -2 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(-2)$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(7) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 28 & -9 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(+3) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 28 & -9 & | & 8 \\ 0 & 0 & 21 & -9 & | & 7 \end{bmatrix} :4 :3 \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 7 & -\frac{9}{4} & | & 2 \\ 0 & 0 & 7 & -3 & | & \frac{7}{3} \end{bmatrix} \cdot(-1)$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 7 & -\frac{9}{4} & | & 2 \\ 0 & 0 & 0 & -\frac{3}{4} & | & -\frac{14}{3} \end{bmatrix}$$

Iz IV vrsta

$$-\frac{3}{4}u = -\frac{14}{3}$$

$$u = \frac{56}{9} \approx 6,2$$

Iz III vrsta

$$7z - \frac{9}{4} \cdot \frac{56}{9} = 2$$

$$7z - 14 = 2$$

$$7z = 16$$

$$z = \frac{16}{7} \approx 2,28$$

Iz II vrsta

$$y + 5z - u = 0$$

$$x_2 + \frac{16}{7} \cdot 5 - \frac{56}{9} = 0$$

$$x_2 = -\frac{328}{63} \approx -5,20..$$

Iz I vrsta

$$x_1 + 2y - 3z + u = -1$$

$$x + 2 \cdot \frac{328}{63} - 3 \cdot \frac{16}{7} + \frac{56}{9} = -1$$

$$x + \frac{88}{9} = -1$$

$$x = -1 - \frac{88}{9}$$

$$x = -\frac{97}{9}$$

$$\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} -\frac{97}{9} \\ \frac{328}{63} \\ \frac{16}{7} \\ \frac{56}{9} \end{bmatrix}$$

5. konveksnost, konkavnost, točke infleksije

$$f''(x) = \frac{(2x-1)\cdot(x-1)^2 - (x^2-2x-3)\cdot 2(x-1)}{(x-1)^4}$$

$$= \frac{(x-1)[(2x-1)(x-1) - 2(x^2-2x-3)]}{(x-1)^{3}}$$

$$= \frac{2x^2 - 2x - x + 1 - 2x^2 + 4x + 6}{(x-1)^3}$$

$$= \frac{2x+7}{(x-1)^3}$$

$$\frac{2x+7}{(x-1)^3} = 0 \quad / \cdot (x-1)^3$$

$$2x + 7 = 0$$

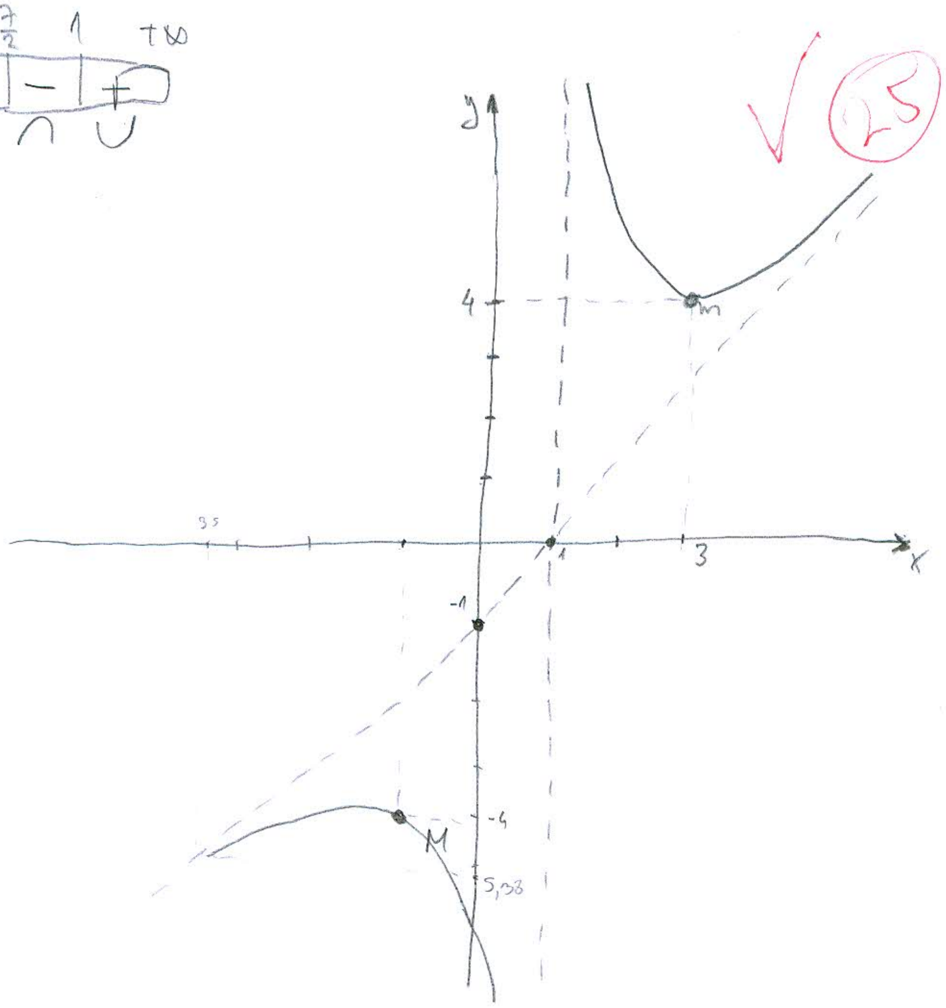
$$2x = -7$$

$$x = -\frac{7}{2} \quad \text{t.I.}$$

?

	$-\infty$	$-\frac{7}{2}$	1	$+\infty$
$f''(x)$	+	−	+	
	∪	∩	∪	

−3,5 , −5,38



MATEO BOBAČEK

♣A

(5) $f(x)=\frac{x^2-2x+5}{x-1}$

1° Domena $x-1\neq 0$, $x\neq 1$, $D(f)=\mathbb{R}\setminus\{1\}$

2° nije periodična

$f(-x)=\frac{(-x)^2-2\cdot(-x)+5}{-x-1}$

$=-\frac{x^2+2x+5}{x+1}$

ni parna ni neparna

NT(,0) s osi x

$f(x)=0$ $\frac{x^2-2x+5}{x-1}=0 \;/\cdot(x-1)$

$x^2-2x+5=0$

ne siječe x os $x_{1,2}=\frac{2\pm\sqrt{4-20}}{2}$

S(0,-5) $f(0)=\frac{0^2-2\cdot 0+5}{0-1}$

$=-\frac{5}{1}=-5$

3° asimptote

VA. $\lim_{x\to 1^-}\frac{x^2-2x+5}{x-1}=\frac{4}{0^-}=-\infty$

$\lim_{x\to 1^+}\frac{x^2-2x+5}{x-1}=\frac{4}{0^+}=+\infty$

VA... $x=1$

HA. $\lim_{x\to\infty}\frac{x^2-2x+5}{x-1}\,/x^2=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{1-\frac{2}{x}+\frac{5}{x^2}}{\frac{1}{x}-\frac{1}{x^2}}=\frac{1}{0}=\infty$ nema HA.

KA $k=\lim_{x\to\infty}\frac{\frac{x^2-2x+5}{x-1}}{x}=\frac{x^2-2x+5}{x^2-x}\,/x^2=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{1-\frac{2}{x}+\frac{5}{x^2}}{1-\frac{1}{x}}=\frac{1}{1}=1$

$l=\lim_{x\to\infty}\frac{x^2-2x+5}{x-1}-x=\lim_{x\to\infty}\frac{x^2-2x+5-x^2+x}{x-1}=\lim_{x\to\infty}\frac{-x+5}{x-1}\,/x=\left[\frac{\infty}{\infty}\right]$

$\lim_{x\to\infty}\frac{-\frac{1}{x}+\frac{5}{x^2}}{1-\frac{1}{x}}=\frac{-1}{1}=-1$

KA... $y=x-1$

x	0	1
y	-1	0

4° ekstremi i monotonost i stacijonarne točke

$f'(x)=\frac{(2x-2)\cdot(x-1)-(x^2-2x+5)\cdot 1}{(x-1)^2}$

$f'(x)=\frac{2x^2-2x-2x+2-x^2+2x-5}{(x-1)^2}$

$f'(x)=\frac{x^2-2x-3}{(x-1)^2}$

$\frac{x^2-2x-3}{(x-1)^2}=0 \;/\cdot(x-1)^2$

$x^2-2x-3=0$

$x_{1,2}=\frac{2\pm\sqrt{4+12}}{2}$

$x_{1,2}=\frac{2\pm 4}{2}$

$x_1=3$ $x_2=-1$

	$-\infty$		-1		1		3		$+\infty$
$f'(x)$		+	0	−	∄	−	0	+	
		↗	M	↘		↘	m	↗	

M(-1,-4) $f(-1)=-4$

m(3,4) $f(3)=4$ ✓

45/100

MATEMATIKA 1
7. veljače 2013.

Ime i prezime: RINO KURTIN **Broj indeksa:** 17-2-0112-2011

Vrijeme: od 08:00 **do** 10:00 **♣A** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Riješi sljedeći sustav jednadžbi:

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5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x)=\frac{x^2-2x+5}{x-1}$$

3. $f(x) = \frac{3x+1}{(x-1)^2}$ $\quad D_f = \mathbb{R} \setminus \{1\}$

$(x-1)^2 \neq 0$

$x^2 - 2x + 1 \neq 0$

$x \neq 1$

V.A

$\lim_{x \to 1^-} \frac{3x+1}{(x-1)^2} = \lim_{x \to 1^-} \frac{4}{0^-} = -\infty$

$\lim_{x \to 1^+} \frac{4}{0^+} = +\infty$ $\quad x = 1$ V.A

H.A

$\lim_{x \to +\infty} \frac{3x+1}{x^2-2x+1} = \left[\frac{\infty}{\infty}\right] \begin{matrix} :x^2 \\ :2 \end{matrix} = \frac{0}{1} = 0$

15

$\lim_{x \to -\infty} \frac{0}{1} = 0$ $\quad y = 0$ H.A

KOSE NEMA

1.

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1 \end{matrix} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & -7 & 7 & -2 & 8 \\ 0 & -3 & 6 & -6 & 7 \end{array}\right] \begin{matrix} R_1 - 2R_2 \\ R_3 + 7R_2 \\ R_4 + 3R_2 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -13 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 42 & -2 & 8 \\ 0 & 0 & 21 & -6 & 7 \end{array}\right] \begin{matrix} \\ \\ :42 \\ \end{matrix} \left[\begin{array}{cccc|c} 1 & 0 & -13 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -1/21 & \frac{4}{21} \\ 0 & 0 & 21 & -6 & 7 \end{array}\right] \begin{matrix} R_1 + 13R_3 \\ R_2 - 5R_3 \\ R_4 - 21R_3 \end{matrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 8/21 & 31/21 \\ 0 & 1 & 0 & 5/21 & -20/21 \\ 0 & 0 & 1 & -1/21 & 4/21 \\ 0 & 0 & 0 & -5 & 3 \end{array}\right] \left[\begin{array}{cccc|c} 1 & 0 & 0 & 8/21 & 31/21 \\ 0 & 1 & 0 & 5/21 & -20/21 \\ 0 & 0 & 1 & -1/21 & 4/21 \\ 0 & 0 & 0 & 1 & -\frac{3}{5} \end{array}\right] \begin{matrix} R_1 - 8/21 R_4 \\ R_2 - 5/21 R_4 \\ R_3 + \frac{1}{21} R_4 \end{matrix}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

4. a) $f(x) = \frac{1}{\ln(x-3)}$

$(\ln(x-3))' = \frac{1}{(x-3)} \cdot (x-3)'$

$f'(x) = \frac{1' \cdot (\ln(x-3)) - 1 \cdot (\ln(x-3))'}{(\ln(x-3))^2} = \frac{-\frac{1}{x-3}}{(\ln(x-3))^2} = \frac{1}{x-3} \cdot 1$

12.5

b) $f(x) = \sqrt{\ln\left(\frac{2x-1}{2x+7}\right)}$

$\frac{2x-1}{2x+7} \geq 0 \;/$

$2x+7 \neq 0$

$2x \neq -7 \;/:2$

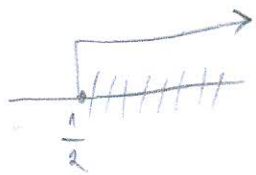
$x \neq -\frac{7}{2}$

$Df = \left[\frac{1}{2}, +\infty\right)$

$2x-1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$



1. $\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1 \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & -7 & 7 & -2 & 8 \\ 0 & -3 & 6 & -6 & 7 \end{array}\right] \begin{array}{l} R_1 - 2R_2 \\ R_3 + 7R_2 \\ R_4 + 3R_2 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & -13 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 42 & -2 & 8 \\ 0 & 0 & 21 & -6 & 7 \end{array}\right] \cdot 1/42$

$\left[\begin{array}{cccc|c} 1 & 0 & -13 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -1/21 & 4/21 \\ 0 & 0 & 21 & -6 & 7 \end{array}\right] \begin{array}{l} R_1 + 13R_3 \\ R_2 - 5R_3 \\ \\ R_4 - 21R_3 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 8/21 & 31/21 \\ 0 & 1 & 0 & 5/21 & -20/21 \\ 0 & 0 & 1 & -1/21 & 4/21 \\ 0 & 0 & 0 & -5 & 3 \end{array}\right] \cdot \left(-\frac{1}{5}\right)$

12.7

$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 8/21 & 31/21 \\ 0 & 1 & 0 & 5/21 & -20/21 \\ 0 & 0 & 1 & -1/21 & 4/21 \\ 0 & 0 & 0 & 1 & -\frac{3}{5} \end{array}\right] \begin{array}{l} R_1 - \frac{8}{21}R_4 \\ R_2 - \frac{5}{21}R_4 \\ R_3 + \frac{1}{21}R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{179}{105} \\ 0 & 1 & 0 & 0 & -\frac{17}{21} \\ 0 & 0 & 1 & 0 & \frac{17}{105} \\ 0 & 0 & 0 & 1 & -\frac{3}{5} \end{array}\right]$

♣A

5. $f(x) = \dfrac{x^2-2x+5}{x-1}$

$D_f = \mathbb{R} \setminus \{1\}$

$x-1 \neq 0$

$x \neq 1$

V.A

$\lim\limits_{x \to 1^-} \dfrac{x^2-2x+5}{x-1} = \lim\limits_{x \to 1^-} \dfrac{1^2-2\cdot 1+5}{1^- - 1} = \dfrac{4}{0^-} = -\infty$

$\lim\limits_{x \to 1^+} \dfrac{x^2-2x+5}{x-1} = \dfrac{4}{0^+} = +\infty$

$x = 1$ V.A

H.A

$\lim\limits_{x \to +\infty} \dfrac{x^2-2x+5 \,/ : x^2}{x-1 \,/ : x^2} = \dfrac{1}{0} = +\infty$ NEMA H.A

$\lim\limits_{x \to -\infty} \dfrac{x^2-2x+5 \,/ : x^2}{x-1 \,/ : x^2} = \dfrac{1}{0} = +\infty$

15|10

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: IVAN BANOVAC **Broj indeksa:** 17-1-0048-2010 0414

Vrijeme: od 20:25 **do** 10:55 **♣A** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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$$f(x) = \sqrt{\ln(\frac{2x-1}{2x+7})}$$

USMENI
PROT
UGLEDIĆ

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \frac{x^2 - 2x + 5}{x - 1}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 2 & 5 & -1 & 2 & -2 \\ 3 & -1 & -2 & 1 & 5 \\ 1 & -1 & 3 & -5 & 6 \end{array}\right] \begin{matrix} \\ -2R1 \\ -3R1 \\ -R1 \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & -7 & 7 & -2 & 8 \\ 0 & -3 & 6 & -6 & 7 \end{array}\right] \begin{matrix} \\ \\ +7R2 \\ +3R2 \end{matrix} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 42 & -2 & 8 \\ 0 & 0 & 21 & -6 & 7 \end{array}\right] \begin{matrix} \\ \\ \\ -\frac{1}{2}R3 \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 42 & -2 & 8 \\ 0 & 0 & 0 & -5 & 3 \end{array}\right] \begin{matrix} \\ \\ :42 \\ :5 \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{21} & \frac{4}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right] \begin{matrix} \\ \\ \\ +\frac{1}{21}R4 \end{matrix} \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{20}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right] -5R3 \sim \left[\begin{array}{cccc|c} 1 & 2 & -3 & 1 & -1 \\ 0 & 1 & 0 & 0 & -\frac{20}{21} \\ 0 & 0 & 1 & 0 & \frac{5}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right] -2R2 \sim \left[\begin{array}{cccc|c} 1 & 0 & -3 & 1 & -\frac{61}{21} \\ 0 & 1 & 0 & 0 & -\frac{20}{21} \\ 0 & 0 & 1 & 0 & \frac{5}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right] +3R3 \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & -\frac{46}{21} \\ 0 & 1 & 0 & 0 & -\frac{20}{21} \\ 0 & 0 & 1 & 0 & \frac{5}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right] -1R4 \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{167}{105} \\ 0 & 1 & 0 & 0 & -\frac{20}{21} \\ 0 & 0 & 1 & 0 & \frac{5}{21} \\ 0 & 0 & 0 & 1 & \frac{3}{5} \end{array}\right]$$

$$-1 - \frac{40}{21}$$

$$\frac{-21-40}{21} = -\frac{61}{21}$$

$$x + 2y - 3z + w = -1$$

$$-1{,}6 + (-1{,}9) - [illegible] 0{,}7 + 0{,}6 = -1$$

$$\frac{-61}{21} + \frac{15}{21} = \frac{-46}{21}$$

$$-\frac{46}{21} + \frac{3}{5} = \frac{-230+63}{105} = \frac{-167}{105}$$

$$\frac{20}{105} + \frac{3}{105} = \frac{23}{105} \qquad \frac{3}{5} \cdot \frac{1}{21} \; \frac{3}{105}$$

$$\frac{4}{21} + \frac{3}{105} = \frac{17}{105}$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 2 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 1 & -1 & 3 & -5 & | & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 1 & -1 & 3 & -5 & | & 6 \\ 3 & -1 & -2 & 1 & | & 5 \\ 2 & 5 & -1 & 2 & | & -2 \end{bmatrix} \begin{matrix} \\ -R1 \\ -3R1 \\ -2R1 \end{matrix} \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & -3 & 6 & -6 & | & 7 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & 1 & 5 & 0 & | & 10 \end{bmatrix} \begin{matrix} \\ :-3 \\ :-7 \\ \\ \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{7}{3} \\ 0 & 1 & -1 & \frac{2}{7} & | & \frac{8}{7} \\ 0 & 1 & 5 & 0 & | & 10 \end{bmatrix} \begin{matrix} \\ \\ \\ -R4 \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{7}{3} \\ 0 & 0 & -6 & -\frac{3}{7} & | & -\frac{8}{7} \\ 0 & 1 & 5 & 0 & | & 10 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & | & \frac{7}{3} \\ 0 & 0 & -6 & -\frac{3}{7} & | & -\frac{8}{7} \\ 0 & 1 & \frac{1}{2} & \frac{1}{7} & | & \frac{3}{3} \end{bmatrix} \begin{matrix} \\ \\ \cdot -1 \\ -R2 \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & 1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & 0 & 6 & \frac{3}{7} & | & \frac{8}{7} \\ 0 & 0 & -\frac{11}{2} & \frac{1}{2} & | & \frac{7}{3} \end{bmatrix} \begin{matrix} \\ \\ :6 \\ :-\frac{11}{2} \end{matrix} \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & 1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & 0 & 1 & \frac{1}{21} & | & \frac{4}{21} \\ 0 & 0 & 1 & \frac{1}{11} & | & \frac{14}{33} \end{bmatrix} \begin{matrix} \\ \\ \\ -R3 \end{matrix} \sim$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & 1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & 0 & 1 & \frac{1}{21} & | & \frac{4}{21} \\ 0 & 0 & 0 & \frac{10}{231} & | & \end{bmatrix}$$

$$-\frac{1}{2} - \frac{5}{1} = -\frac{11}{2} \qquad \frac{1}{2} \quad \frac{2}{+22} \quad \frac{1}{11}$$

$$\frac{2}{7} \quad \frac{2}{42} = \frac{1}{21}$$

$$\frac{2}{6} \quad \frac{1}{1}$$

$$\frac{8}{7} \quad \frac{8}{42} = \frac{4}{21}$$

$$\frac{6}{1}$$

$$\frac{2}{21} \quad \frac{1}{21}$$

$$\frac{21}{231} = \frac{10}{231}$$

$$\frac{4}{693}$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 2 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 1 & -1 & 3 & -5 & | & 6 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \\ R_4 - R_1 \end{matrix} \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \begin{matrix} 2R_2 \\ \sim \\ R_3 + 7R_2 \\ R_4 + 3R_2 \end{matrix} \begin{bmatrix} 1 & 0 & -13 & 1 & | & -1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & 0 & 42 & -2 & | & 8 \\ 0 & 0 & 21 & -6 & | & 7 \end{bmatrix} \begin{matrix} \\ \\ \\ :2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -13 & 1 & | & -1 \\ 0 & 1 & 5 & 0 & | & 0 \\ 0 & 0 & 1 & -\frac{1}{21} & | & \frac{4}{21} \\ 0 & 0 & 21 & -6 & | & 7 \end{bmatrix} \begin{matrix} R_1 + 13R_3 \\ R_2 - 5R_3 \\ \\ R_4 - 21R_3 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{8}{21} & | & \frac{31}{21} \\ 0 & 1 & 0 & \frac{5}{21} & | & -\frac{20}{21} \\ 0 & 0 & 1 & -\frac{1}{21} & | & \frac{4}{21} \\ 0 & 0 & 0 & -5 & | & 3 \end{bmatrix} \cdot \left(-\frac{1}{5}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{8}{21} & | & \frac{31}{21} \\ 0 & 1 & 0 & \frac{5}{21} & | & -\frac{20}{21} \\ 0 & 0 & 1 & -\frac{1}{21} & | & \frac{4}{21} \\ 0 & 0 & 0 & 1 & | & -\frac{3}{5} \end{bmatrix} \begin{matrix} R_1 - \frac{8}{21} R_4 \\ R_2 - \frac{5}{21} R_4 \\ R_3 + \frac{1}{21} R_4 \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & | & \frac{179}{105} \\ 0 & 1 & 0 & 0 & | & -\frac{17}{21} \\ 0 & 0 & 1 & 0 & | & \frac{17}{105} \\ 0 & 0 & 0 & 1 & | & -\frac{3}{5} \end{bmatrix}$$

$$x = \frac{179}{105} \qquad y = -\frac{17}{21} \qquad z = \frac{17}{105}$$

$$u = -\frac{3}{5}$$

5.

$$f(x) = \frac{x^2 - 2x + 5}{x - 1} \qquad Df \langle -\infty\ 1) \cup (1\ +\infty \rangle$$

$$x - 1 \neq 0$$
$$x \neq 1$$

$$\lim_{x \to 1} = \frac{1^2 - 2 + 5}{1 - 1} = \frac{4}{0} = \infty$$ VERTIKALNA ASIMP

$$\lim_{x \to \infty} \overset{LH}{=} \frac{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{5}{x^2}}{\frac{x}{x^2} - 1} = \frac{1}{-1} = 1$$

$$\underset{x \to}{KA} =$$

3. $f(x) = \frac{3x+1}{(x-1)^2}$

$(x-1)^2 \neq 0$

$x-1 \neq 0$

$x \neq 1$

$\lim_{x\to 1} \frac{3x+1}{(x-1)^2} \overset{\ln}{=} \frac{4}{0} = \infty$ $\quad x \to 1$ vertikalna asimptota

$\lim_{x\to 1^+} \frac{4}{0^+} = +\infty$

$\lim_{x\to 1^-} \frac{4}{0^-} = -\infty$

U (15)

$\lim$

$\lim_{x\to\infty} \frac{3x+1}{(x-1)^2} \overset{\infty}{=} \overset{LH}{\lim_{x\to 1}} \frac{3}{2(x-1)+(x-1)'} = \frac{3}{2x-2+1} = \frac{3}{2x-1} = \frac{3}{\infty} = 0$

$\lim_{x\to -\infty} \frac{3}{-\infty} = 0$

$y=0$ Horizontalna asimptota

OBOSTRANA

4. $f'(x) = \frac{1}{\ln(x-3)} = \frac{1'\cdot(\ln(x-3)) - 1(\ln(x-3))'}{\ln^2(x-3)} = \frac{1\cdot\left(\frac{1}{x-3}\cdot(x-3)'\right)}{\ln^2(x-3)} = \frac{1\cdot\frac{1}{x-3}}{\ln^2(x-3)} = \frac{\frac{1}{x-3}}{\ln^2 x - 3} = \frac{1}{x\ln^2 x - 3\ln^2 x}$

$f(x) \sqrt{\ln\left(\frac{2x-1}{2x+7}\right)}$

$2x+7 \neq 0$

$2x \neq -7$

$x \neq \frac{-7}{2}$

$\ln\left(\frac{2x-1}{2x+7}\right) \geq 0$

$\frac{2x-1}{2x+7} \geq 1$

$\frac{2x-1}{2x+7} - 1 > 0$

$\frac{2x-1-2x-7}{2x+7} = \frac{-8}{2x+7} > 0$

$D = \left\langle -\infty, \frac{-7}{2}\right\rangle \cup \left\langle \frac{1}{2}, \infty\right\rangle$

$2x+7 < 0$

$2x < -7$

$x < \frac{-7}{2}$

$\frac{2x+1}{2x+7} > 0$

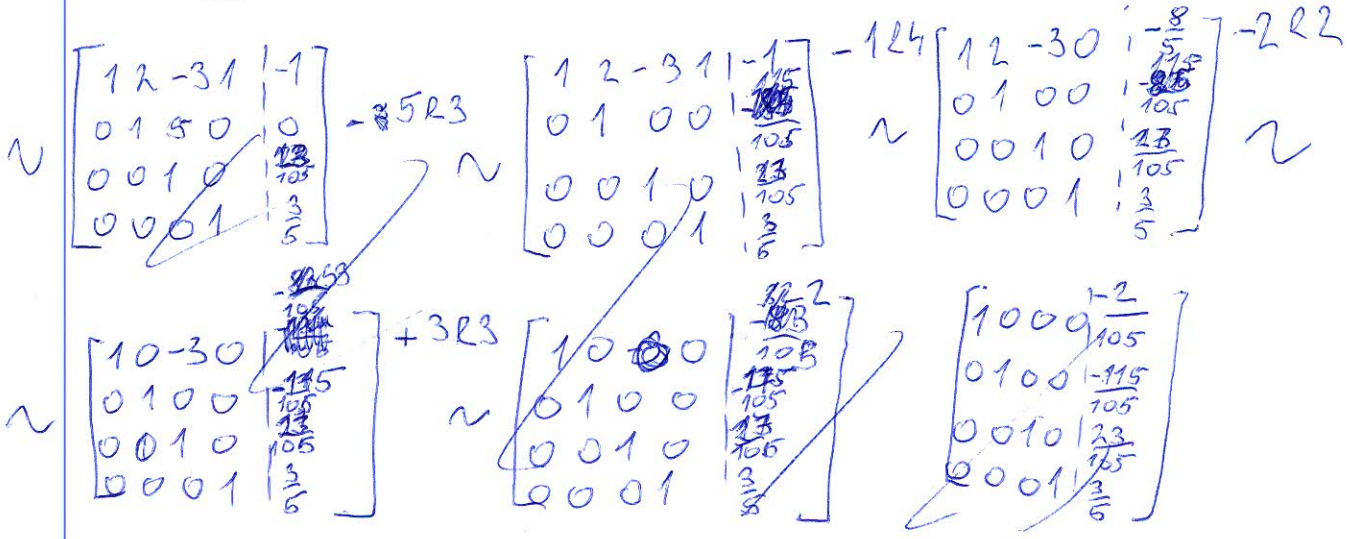
$2x+1 = 0 \quad x = \frac{-1}{2}$

$2x+7 = 0 \quad x = \frac{-7}{2}$

$x \geq 0$

	$-\infty$		$\frac{-7}{2}$		$\frac{-1}{2}$		∞
$2x+7$	−	0	+		+		+
$2x-1$	+		−		0		+
	+	∄	−		0		+

♣A



$$-\frac{8}{5}+\frac{34}{105}=\frac{134}{105} \qquad -\frac{8}{5}+\frac{85}{105}=\frac{-83}{105}$$

$$\frac{-134}{105}+\frac{51}{105}=\frac{-83}{105} \qquad -\frac{8}{5}+\frac{115}{105}$$

$$X+2Y-3Z+1U=-1$$

$$-0,8-0,08-0,16+0,6=-1$$

$$0,02-1,00-0,12+0,6=-1$$

$$-\frac{83}{100}+\frac{51}{105}=\frac{-32}{105}$$

$$\frac{5}{1}-\frac{23}{105}$$

$$-\frac{53}{105}+\frac{51}{105}=\frac{-2}{105}$$