52.5|100

KOSOR

MATEMATIKA 1

7. veljače 2013.

Ime i prezime: MATEO BOBAČEK **Broj indeksa:** 17-2-0113-2011

Vrijeme: od_____ **do**_____ **♣A** **Broj bodova:**

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(17.5)* Riješi sljedeći sustav jednadžbi:

$$\begin{aligned} x + 2y - 3z + u &= -1 \\ 2x + 5y - z + 2u &= -2 \\ 3x - y - 2z + u &= 5 \\ x - y + 3z - 5u &= 6 \end{aligned}$$

2. *(17.5)* Riješi u skupu $\mathbb{C}$ jednadžbu:

$$z^2 + \frac{(1-i)^3}{i^{25}} = 0$$

3. *(15)* Odredi asimptote sljedeće funkcije:

$$f(x) = \frac{3x+1}{(x-1)^2}$$

4. *(12.5+12.5)*

a) Deriviraj funkciju:

$$f(x) = \frac{1}{\ln(x-3)}$$

b) Odredi domenu funkcije:

$$f(x) = \sqrt{\ln(\frac{2x-1}{2x+7})}$$

5. *(25)* Ispitaj tok i skiciraj graf funkcije:

$$f(x) = \frac{x^2 - 2x + 5}{x - 1}$$

② $z^2 + \frac{(1-i)^3}{i^{25}} =$

$z^2 = -\frac{(1-i)^3}{i} \cdot \frac{i}{i}$

$z^2 = -\frac{i(1-i)^3}{i^2}$

$z^2 = -\frac{i(1-i)^3}{-1}$

$z^2 = i(1-i)^3$

6

③ $f(x) = \frac{3x+1}{(x-1)^2} = \frac{3x+1}{x^2-2x-1}$

Domena

$(x-1)^2 \neq 0$ /$\sqrt{\ }$

$x-1 \neq$

$x \neq 1$ $\quad D(f) = \mathbb{R} \setminus \{1\}$

VA. $\lim_{x \to 1^-} \frac{3x+1}{(x-1)^2} = \frac{4}{0^-} = +\infty$

$\lim_{x \to 1^+} \frac{3x+1}{(x-1)^2} = \frac{4}{0^+} = +\infty$

VA... $x = 1$

15

HA. $\lim_{x \to \infty} \frac{3x+1}{(x-1)^2} = \lim_{x \to \infty} \frac{3x+1 /:x^2}{x^2-2x+1 /:x^2} = \lim_{x \to \infty} \frac{\frac{3}{x} + \frac{1}{x^2}}{1 - \frac{2}{x} + \frac{1}{x^2}} = \frac{0}{1} = 0$ $\quad$ HA... $y = 0$

K.A. nema

④ a) $f(x) = \frac{1}{\ln(x-3)} = (\ln(x-3))^{-1}$

$f'(x) = -1 \cdot (\ln(x-3))^{-2} \cdot \frac{1}{x-3} \cdot 1$

$= -(\ln(x-3))^{-2} \cdot \frac{1}{x-3}$ //

12.5

b) $f(x) = \sqrt{\ln\left(\frac{2x-1}{2x+7}\right)}$

$\ln\left(\frac{2x-1}{2x+7}\right) \geq 0 / \cdot \exp^e$

$\frac{2x-1}{2x+7} \geq 1$

$\frac{2x-1}{2x+7} - 1 \geq 0$

$\frac{2x-1-2x-7}{2x+7} \geq 0$

$\frac{-8}{2x+7} \geq 0$

$2x+7 \geq 0$

$2x \geq -7$

$\frac{2x-1}{2x+7} > 0$

$x = \frac{1}{2}$ $\quad x = -\frac{7}{2}$

$2x+7 \neq 0$

$2x \neq -7$

$x \neq -\frac{7}{2}$

	$-\infty$		$\frac{1}{2}$		$+\infty$
$2x-1$		−	0	+	

$D(f) = \langle \frac{1}{2}, +\infty \rangle \setminus \{-\frac{7}{2}\}$

MATEO BOBAČEK

1.

♣2 A

$$\begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 2 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 1 & -1 & 3 & -5 & | & 6 \end{bmatrix} \cdot(-1) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 1 & | & -2 \\ 3 & -1 & -2 & 1 & | & 5 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(-3) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 2 & 5 & -1 & 1 & | & -2 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(-2)$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & -7 & 7 & -2 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(7) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 28 & -9 & | & 8 \\ 0 & -3 & 6 & -6 & | & 7 \end{bmatrix} \cdot(+3) \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 28 & -9 & | & 8 \\ 0 & 0 & 21 & -9 & | & 7 \end{bmatrix} :4 :3 \sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 7 & -\frac{9}{4} & | & 2 \\ 0 & 0 & 7 & -3 & | & \frac{7}{3} \end{bmatrix} \cdot(-1)$$

$$\sim \begin{bmatrix} 1 & 2 & -3 & 1 & | & -1 \\ 0 & 1 & 5 & -1 & | & 0 \\ 0 & 0 & 7 & -\frac{9}{4} & | & 2 \\ 0 & 0 & 0 & -\frac{3}{4} & | & -\frac{14}{3} \end{bmatrix}$$

Iz IV vrsta

$$-\frac{3}{4}u = -\frac{14}{3}$$

$$u = \frac{56}{9} \approx 6,2$$

Iz III vrsta

$$7z - \frac{9}{4} \cdot \frac{56}{9} = 2$$

$$7z - 14 = 2$$

$$7z = 16$$

$$z = \frac{16}{7} \approx 2,28$$

Iz II vrsta

$$y + 5z - u = 0$$

$$x_2 + \frac{16}{7} \cdot 5 - \frac{56}{9} = 0$$

$$x_2 = -\frac{328}{63} \approx -5,20..$$

Iz I vrsta

$$x_1 + 2y - 3z + u = -1$$

$$x + 2 \cdot \frac{328}{63} - 3 \cdot \frac{16}{7} + \frac{56}{9} = -1$$

$$x + \frac{88}{9} = -1$$

$$x = -1 - \frac{88}{9}$$

$$x = -\frac{97}{9}$$

$$\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} -\frac{97}{9} \\ \frac{328}{63} \\ \frac{16}{7} \\ \frac{56}{9} \end{bmatrix}$$

5. konveksnost, konkavnost, točke infleksije

$$f'(x)=\frac{(2x-1)\cdot(x-1)^2-(x^2-2x-3)\cdot 2(x-1)}{(x-1)^4}$$

$$=\frac{(x-1)\left[(2x-1)(x-1)-2(x^2-2x-3)\right]}{(x-1)^{\cancel{4}3}}$$

$$=\frac{2x^2-2x-x+1-2x^2+4x+6}{(x-1)^3}$$

$$=\frac{2x+7}{(x-1)^3}$$

$$\frac{2x+7}{(x-1)^3}=0 \quad /\cdot(x-1)^3$$

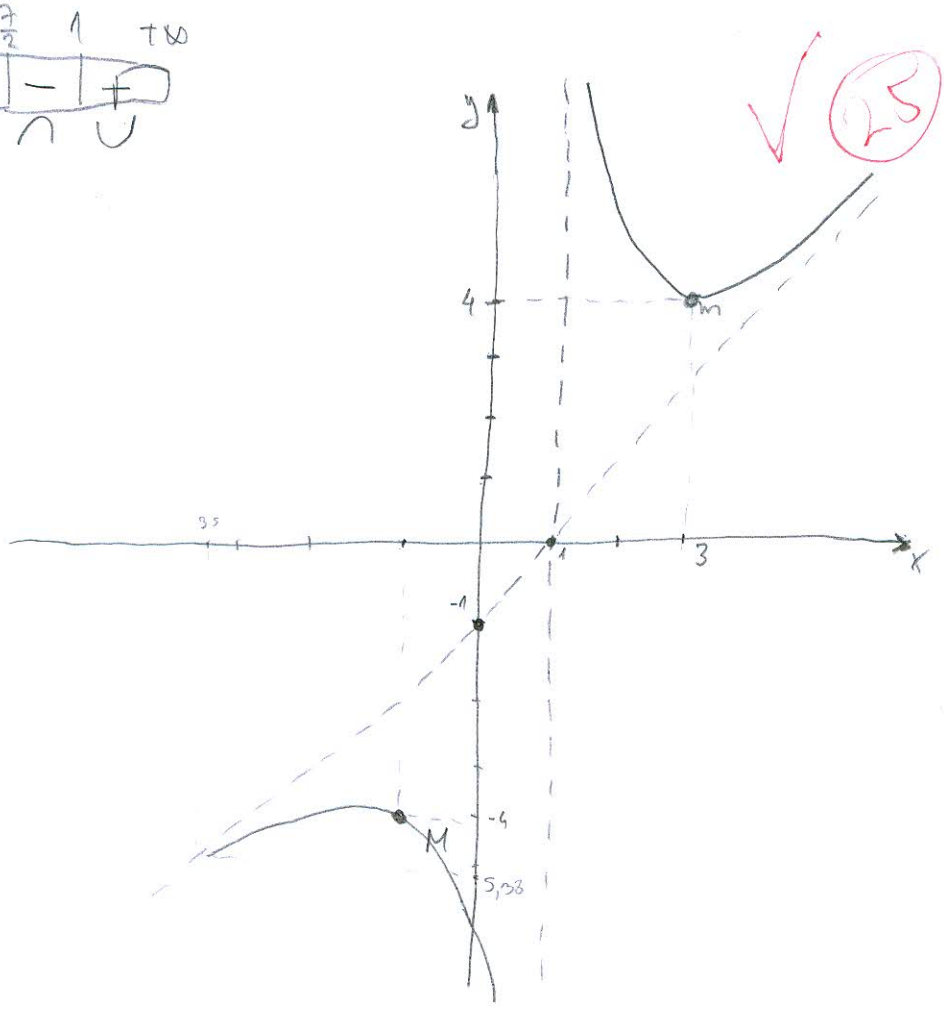
$$2x+7=0$$

$$2x=-7$$

$$x=-\frac{7}{2}$$ t.i.

	$-\infty$		$-\frac{7}{2}$		1		$+\infty$
$f''(x)$		$+$		$-$		$+$	
		$\cup$		$\cap$		$\cup$	

$-3,5$, $-5,38$



√ 2,5

MATEO BOBAČEK

♣A

(5) $f(x)=\frac{x^2-2x+5}{x-1}$

1° Domena $x-1\neq 0$, $x\neq 1$, $D(f)=\mathbb{R}\setminus\{1\}$

2° nije periodična

$f(-x)=\frac{(-x)^2-2\cdot(-x)+5}{-x-1}$

$=-\frac{x^2+2x+5}{x+1}$

ni parna ni neparna

NT(,0) s osi x

$f(x)=0$ $\frac{x^2-2x+5}{x-1}=0 \;/\cdot(x-1)$

$x^2-2x+5=0$

ne siječe x os $x_{1,2}=\frac{2\pm\sqrt{4-20}}{2}$

S(0,-5) $f(0)=\frac{0^2-2\cdot 0+5}{0-1}$

$=-\frac{5}{1}=-5$

3° asimptote

VA. $\lim_{x\to 1^-}\frac{x^2-2x+5}{x-1}=\frac{4}{0^-}=-\infty$

$\lim_{x\to 1^+}\frac{x^2-2x+5}{x-1}=\frac{4}{0^+}=+\infty$

VA... $x=1$

HA. $\lim_{x\to\infty}\frac{x^2-2x+5}{x-1}\,/x^2=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{1-\frac{2}{x}+\frac{5}{x^2}}{\frac{1}{x}-\frac{1}{x^2}}=\frac{1}{0}=\infty$ nema HA.

KA $k=\lim_{x\to\infty}\frac{\frac{x^2-2x+5}{x-1}}{x}=\frac{x^2-2x+5}{x^2-x}\,/x^2=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{1-\frac{2}{x}+\frac{5}{x^2}}{1-\frac{1}{x}}=\frac{1}{1}=1$

$l=\lim_{x\to\infty}\frac{x^2-2x+5}{x-1}-x=\lim_{x\to\infty}\frac{x^2-2x+5-x^2+x}{x-1}=\lim_{x\to\infty}\frac{-x+5}{x-1}\,/x=\left[\frac{\infty}{\infty}\right]$

$\lim_{x\to\infty}\frac{-\frac{1}{x}+\frac{5}{x^2}}{1-\frac{1}{x}}=\frac{-1}{1}=-1$

KA... $y=x-1$

x	0	1
y	-1	0

4° ekstremi i monotonost i stacijonarne točke

$f'(x)=\frac{(2x-2)\cdot(x-1)-(x^2-2x+5)\cdot 1}{(x-1)^2}$

$f'(x)=\frac{2x^2-2x-2x+2-x^2+2x-5}{(x-1)^2}$

$f'(x)=\frac{x^2-2x-3}{(x-1)^2}$

$\frac{x^2-2x-3}{(x-1)^2}=0 \;/\cdot(x-1)^2$

$x^2-2x-3=0$

$x_{1,2}=\frac{2\pm\sqrt{4+12}}{2}$

$x_{1,2}=\frac{2\pm 4}{2}$

$x_1=3$ $x_2=-1$

	$-\infty$		-1		1		3		$+\infty$
$f'(x)$		+	0	−	∄	−	0	+	
		↗	M	↘		↘	m	↗	

M(-1,-4) $f(-1)=-4$

m(3,4) $f(3)=4$ ✓

45/100

MATEMATIKA 1
7. veljače 2013.

Ime i prezime: RINO KURTIN **Broj indeksa:** 17-2-0112-2011

Vrijeme: od 08:00 **do** 10:00 **♣A** **Broj bodova:**

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