

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: DINO KURIC

BROJ INDEKSA: 56192-2008

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

POPUNJAVA

NASTAVNIK

Broj ↓

bodova

- Zadan trokut T sa vrhovima: $A(1, 1)$, $B(2, 3)$ i $C(3, 2)$ i funkcija $f(x, y) = xy$. Odrediti $\iint_T f(x, y) dx dy$. 20
- Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (3 - 2y) dx dy$. 20
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- Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2 + 3y) dx dy$. 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0.$$

Tablica integrala

Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
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$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

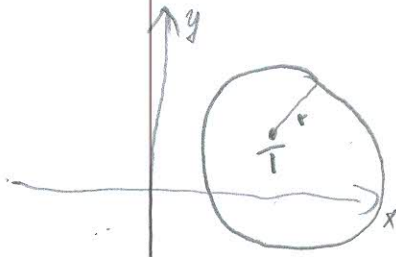
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Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
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t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
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$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



2. $r=1$ $T(2,1)$; $\iint_K (3-2y) dx dy =$



Polarne koordinate

$$x = r \cos \varphi + 2$$

$$y = r \sin \varphi + 1$$

$$r \in (0, 1)$$

$$\varphi \in (0, 2\pi)$$

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2r \sin \varphi) r dr$$

$$\int_0^{2\pi} d\varphi \cdot 4 \int_0^1 -2r \sin \varphi dr$$

$$\int_0^{2\pi} \sin \varphi d\varphi \cdot 4 \int_0^1 -2r \cdot r dr$$

$$4 \int_0^{2\pi} -\cos \varphi d\varphi \cdot 4 \cdot (-2) \cdot \int_0^1 r^2 dr$$

$$= -\cos \varphi \Big|_0^{2\pi} \cdot 8 \cdot \frac{r^3}{3} \Big|_0^1$$

0,33

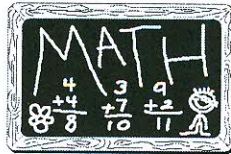
$$= -\cos 2\pi \Big|_0^{2\pi} - 8 \cdot \frac{1}{3} \Big|_0^1$$

$$= -(\cos 2\pi + \cos 0) \Big|_0^{2\pi} - 8 \cdot \frac{1}{3}$$

$$= -1 + 1 -$$

$$= -0,33$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



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IME I PREZIME: *Igor Brajica*

BROJ INDEKSA: *52803-2005*

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

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⑤ $f'''(t) - f'(t) = \cos(t)$ $f(0) = 1$

$f'(0) = 0$

$f''(0) = 0$

$$s^3 F(s) - \overset{1}{s^2} f(0) - \overset{0}{s} f'(0) - \overset{0}{f''(0)} - s F(s) - \overset{1}{f(0)} = \frac{s}{s^2+1}$$

$$s^3 F(s) - s^2 - s F(s) - 1 = \frac{s}{s^2+1}$$

$$F(s) (s^3 - s) = \frac{s}{s^2+1} + s^2 + 1$$

$$F(s) (s^3 - s) = \frac{s + s^4 + s^2 + s^2 + 1}{s^2+1}$$

$$F(s) (s^3 - s) = \frac{s^4 + 2s^2 + s + 1}{(s^2+1)(s^3-s)}$$

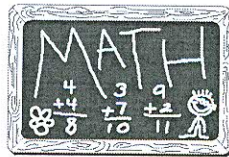
$$F(s) = \frac{s^4 + 2s^2 + s + 1}{s(s^2-1)(s^2+1)}$$

$$F(s) = \frac{s^4 + 2s^2 + s + 1}{s(s-1)(s+1)(s^2+1)}$$

$$s^4 + 2s^2 + s + 1 = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+1} + \frac{Ds + E}{s^2+1}$$

$$s^4 + 2s^2 + s + 1 = A(s^2-1)(s^2+1) + Bs(s+1)(s^2+1) + Cs(s-1)(s^2+1) + Ds^2 + Es(s^2-1)$$

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IME I PREZIME: 52807-2005

BROJ INDEKSA:

$$s^4 + 2s^2 + s + 1 = A(s^4 + s^2 - s^2 - 1) + Bs(s^3 + s + s^2 + 1) +$$

$$Cs(s^3 + s - s^2 - 1) + Ds^2(s^2 - 1) + Es(s^2 - 1)$$

$$s^4 + 2s^2 + s + 1 = \underline{As^4} - A + \underline{Bs^4} + \underline{Bs^2} + \underline{Bs^3} + Bs +$$

$$\underline{Cs^4} + \underline{Cs^2} - \underline{Cs^3} - Cs + \underline{Ds^4} - \underline{Ds^2} + \underline{Es^3} - Es$$

$$1 = A + B + C + D \Rightarrow 2 = B + C + D \quad \left. \begin{array}{l} 1 = A + B + C + D \\ 2 = B + C + D \end{array} \right\} +$$

$$0 = B - C + E$$

$$2 = B + C - D$$

$$2 = B + C - D$$

$$4 = 2B + 2E$$

$$1 = B - C - E \quad / \cdot 2$$

$$2 = 2B - 2C - 2E$$

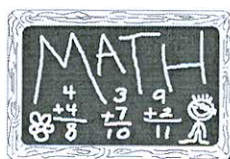
$$6 = 4B - 2E$$

$$\boxed{1 = -A}$$

$$\boxed{A = -1}$$



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IME I PREZIME: **BRUNO LIPOVIĆA**

BROJ INDEKSA: **54960**

VRIJEME POČETKA:

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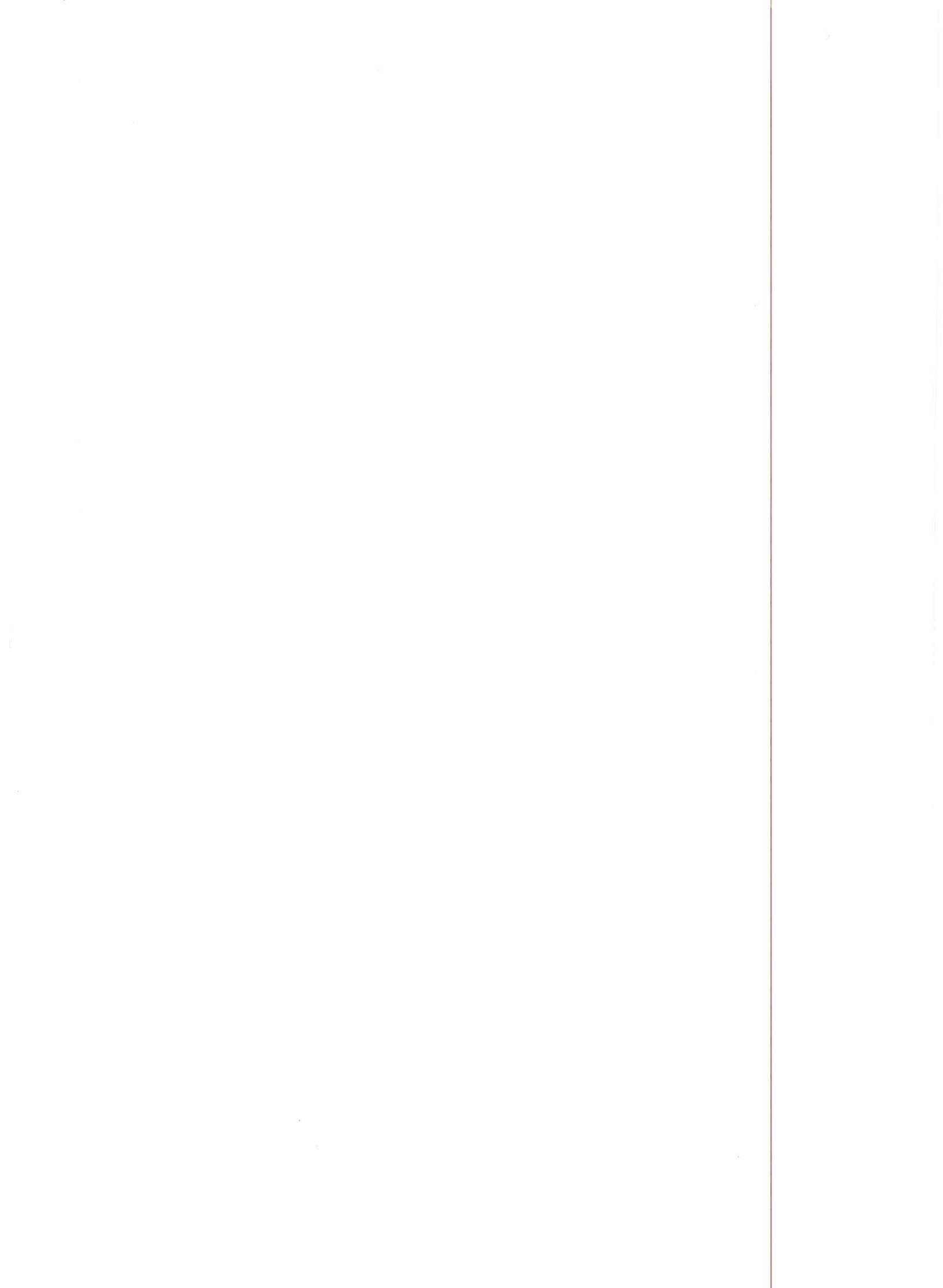
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① $A(x_1, y_1) = (1, 1), B(x_2, y_2) = (2, 3) \quad C(x_3, y_3) = (3, 2)$

\vec{AB}

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{3 - 1}{2 - 1} (x - 1)$$

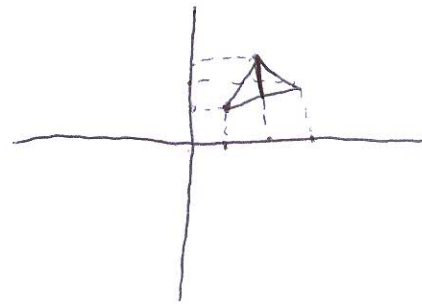
$$y - 1 = \frac{2}{1} (x - 1)$$

$$y - 1 = 2(x - 1)$$

$$y - 1 = 2x - 2$$

$$y = 2x - 2 + 1$$

$$y = 2x - 1$$



BC $y - 3 = \frac{2 - 3}{3 - 2} (x - 2)$

$$y - 3 = \frac{-1}{1} (x - 2)$$

$$y - 3 = -1(x - 2)$$

$$y - 3 = -x + 2$$

$$y = -x + 2 + 3$$

$$y = -x + 5$$

$\vec{AC} \quad y - 1 = \frac{2 - 1}{3 - 1} (x - 1)$

$$y - 1 = \frac{1}{2} (x - 1)$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2} + 1$$

$$y = \frac{1}{2}x + \frac{1}{2}$$



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→ ~~scribble~~

$$P_1 = \int_0^1 dx \int_{\frac{1}{2}x + \frac{1}{2}}^{2+1} dy \quad \times$$

$$\int_0^1 dx \quad 2+1 - \frac{1}{2}x + \frac{1}{2}$$

$$\left. \frac{2x^2}{2} - x - \frac{1}{2} \cdot \frac{x^2}{2} + \frac{1}{2}x \right|_0^1$$

$$2 \cdot \frac{1^2}{2} - 1 - \frac{1}{2} \cdot \frac{1^2}{2} + \frac{1}{2} \cdot 1$$

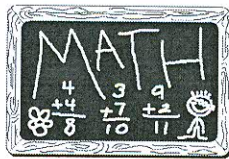
$$\frac{2}{2} - 1 - \frac{1}{2} + \frac{1}{2}$$

$$1 - 1 - \frac{1}{2} + \frac{1}{2}$$

$$= 0$$

$$P_1 + P_2 = 0 + 13 = 13$$

~~scribble~~



$$P_2 = \int_1^3 dx \int_{\frac{1}{2}x + \frac{1}{2}}^{-x+5} dy \quad \times$$

$$P_2 = \int_1^3 -x+5 - \frac{1}{2}x + \frac{1}{2}$$

$$P_2 = \left. -\frac{1}{2}x^2 + 5x - \frac{1}{2} \cdot \frac{x^2}{2} + \frac{1}{2}x \right|_1^3$$

$$P_2 = -\frac{3^2}{2} + 5 \cdot 3 - \frac{1}{2} \cdot \frac{3^2}{2} + \frac{1}{2} \cdot 3$$

$$- \left(-\frac{1^2}{2} + 5 - \frac{1}{2} \cdot \frac{1^2}{2} + \frac{1}{2} \right)$$

$$P_2 = \left(\frac{9}{2} + 15 - \frac{9}{4} + \frac{3}{2} \right) - \left(\frac{1}{4} + 5 - \frac{1}{4} + \frac{1}{2} \right)$$

$$= \left(\frac{9}{2} + 15 - \frac{9}{4} + \frac{3}{2} \right) - \left(\frac{23}{4} \right)$$

$$P_2 = 18.75 - 5.75$$

$$P_2 = 13$$

~~scribble~~

$$\int_0^{2\pi} \int_0^1 (3-2y) \, d\sigma \, dy$$

~~scribble~~

$$d\sigma \, dy = r \, dr \, dt$$

$$y = r \sin \varphi$$

$$\int_0^{2\pi} \int_0^1 (3 - 2r \sin \varphi) r \, dr \, dt$$

$$\int_0^{2\pi} \left(3r \, dr \, dt - 2r^2 \sin \varphi \, dr \, dt \right)$$

$$\int_0^{2\pi} \left. \left(\frac{3r^2}{2} - 2 \frac{r^3}{2} \sin \varphi \right) \right|_0^1$$

$$\int_0^{2\pi} \left(\frac{3r^2}{2} - 2 \frac{r^3}{2} \sin \varphi \right)$$

$$\int_0^{2\pi} \left(\frac{3}{2} - \frac{2}{2} \sin \varphi \right)$$

$$\int_0^{2\pi} \frac{1}{2} \sin \varphi$$

$$\left. \left(\frac{1}{2} - \cos \varphi \right) \right|_0^{2\pi}$$

$$\frac{1}{2} - \cos 2\pi = \left(\frac{1}{2} - \cos 0 \right)$$

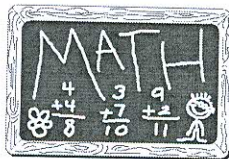
$$\frac{1}{2} - 1 = \left(\frac{1}{2} - 1 \right)$$

$$= -\frac{1}{2} - \left(-\frac{1}{2} \right) = -\frac{1}{2} + \frac{1}{2} = 0$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

③ (x, y, z)

$$\pi(A) = \sqrt{\quad}$$



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

POPUNJAVA
NASTAVNIK
Broj ↓
bodova

IME I PREZIME: **LUKA KURILIĆ**

BROJ INDEKSA: **58076**

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

- Zadan trokut T sa vrhovima: $A(1, 1)$, $B(2, 3)$ i $C(3, 2)$ i funkcija $f(x, y) = xy$. Odrediti $\iint_T f(x, y) dx dy$. 20
- Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (3 - 2y) dx dy$. 20
- Provjeriti da li je krivoljni integral u vektorskom polju $g(x, y, z) = (2x - 1, 3y + z, 2z + y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
- Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2 + 3y) dx dy$. 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0.$$

Tablica integrala

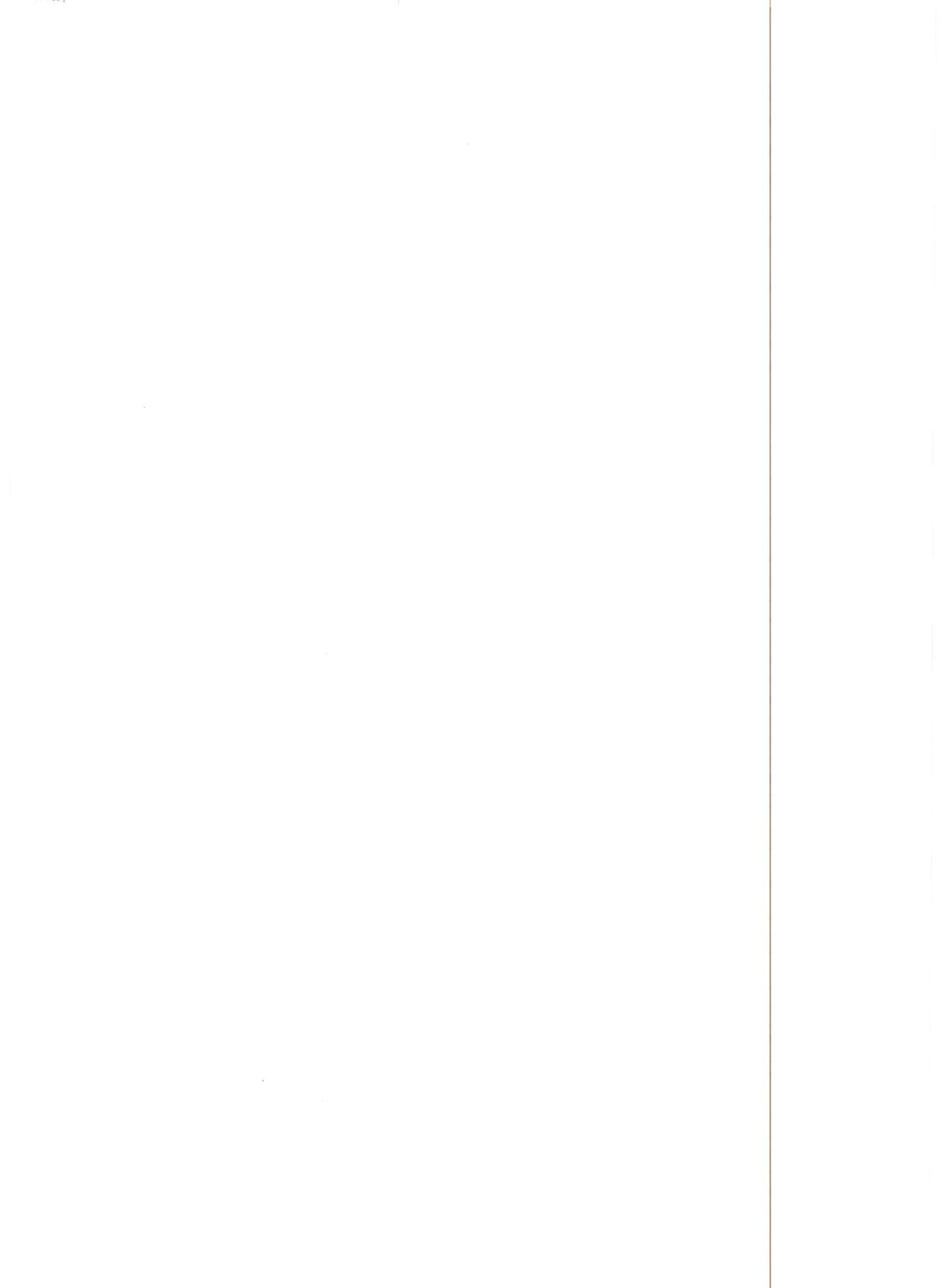
Ukupno: 20

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x \sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

Tablica

Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



$$\textcircled{5} f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0$$

$$f''' \Rightarrow s^3 f(s) - s^2 \underbrace{f(0)}_1 - s \underbrace{f'(0)}_0 - \underbrace{f''(0)}_0$$

$$\boxed{f''' \Rightarrow s^3 f(s) - s^2}$$

$$f' \Rightarrow s f(s) - f(0)$$

$$\boxed{f' \Rightarrow s f(s) - 1}$$

$$s^3 f(s) - s^2 - s f(s) + 1 = \frac{s}{s^2 + 1}$$

$$s^3 f(s) - s f(s) = \frac{s}{s^2 + 1} + \frac{s^2}{1} - \frac{1}{1}$$

$$f(s)(s^3 - s) = \frac{s + (s^2 + 1)s^2 - 1(s^2 + 1)}{s^2 + 1}$$

$$f(s)(s^3 - s) = \frac{s + s^4 + s^2 - s^2 - 1}{s^2 + 1}$$

$$f(s)(s^3 - s) = \frac{s^4 + s - 1}{s^2 + 1}$$

$$f(s) = \frac{s^4 + s - 1}{(s^3 - s)(s^2 + 1)}$$

$$f(s) = \frac{s^4 + s - 1}{s(s^2 - 1)(s^2 + 1)}$$

~~$$\frac{s^4 + s - 1}{s(s^2 - 1)(s^2 + 1)} = \frac{A}{s} + \frac{B+C}{s^2 - 1} + \frac{D+E}{s^2 + 1}$$~~

~~$$\frac{s^4 + s - 1}{s(s^2 - 1)(s^2 + 1)}$$~~

$$s^4 + s - 1 = \frac{A}{s} + \frac{B}{s^2} + \frac{C+D}{s^2 - 1} + \frac{E}{s^2 + 1} \Big/ s(s^2 - 1)(s^2 + 1)$$

$$= A(s^2 - 1)(s^2 + 1) + B(s - 1)(s^2 + 1) + (C + D)(s^2 + 1)s + E(s^2 - 1)s$$

$$= A(s^4 + s^2 - s^2 - 1) + B(s^3 + s - s^2 - 1) + (C + D)(s^3 + s) + E(s^3 - s)$$

$$= \underline{As^4} + \underline{Bs^3} + \underline{Cs^3} + \underline{Ds^3} + \underline{Es^3} - \underline{As^2} - \underline{Bs^2} - \underline{Cs^2} - \underline{Ds^2} - \underline{Es^2} - \underline{A} + \underline{Bs} + \underline{Cs} + \underline{Ds} + \underline{Es} - \underline{E}$$

$$(s^4) \quad 1 = A + C$$

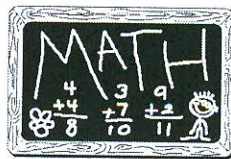
$$(s^3) \quad 0 = B + D + E$$

$$(s^2) \quad 0 = -B + C$$

$$(s^1) \quad 1 = B + D - E$$

$$(s^0) \quad -1 = -A - B$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



5) * Nastavak

~~$$-1 = -A - B$$

$$1 = A + C$$

$$0 = B + D + E$$~~

$$0 = B + D + E$$

$$B = D + E$$

$$B = -\frac{1}{2} + D$$

$$0 = B + D + E$$

$$1 = B + D - E \quad | \cdot (-1)$$

$$0 = B + D + E$$

$$-1 = -B - D + E$$

$$-1 = 2E$$

$$2E = -1$$

$$\boxed{E = -\frac{1}{2}}$$

~~XXXXXXXXXX~~

$$1 = B + D - E$$

$$1 = -\frac{1}{2} + D + D - \frac{1}{2}$$

$$-2D = -\frac{1}{2} - \frac{1}{2} - 1$$

$$-2D = -2$$

$$\boxed{D = 1}$$

$$1 = A + C$$

$$1 = \frac{1}{2} + C$$

$$-C = \frac{1}{2} - 1$$

$$-C = -\frac{1}{2}$$

$$\boxed{C = \frac{1}{2}}$$

$$\rightarrow B = -\frac{1}{2} + 1$$

$$\boxed{B = \frac{1}{2}}$$

~~XXXXXXXXXX~~

$$-1 = -A - B$$

$$-1 = -A - \frac{1}{2}$$

$$A = 1 - \frac{1}{2}$$

$$\boxed{A = \frac{1}{2}}$$

$$A = \frac{1}{2}$$

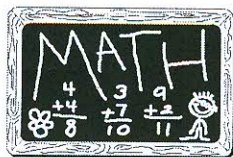
$$B = \frac{1}{2}$$

$$C = \frac{1}{2}$$

$$D = 1$$

$$E = -\frac{1}{2}$$

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(5) **NASTAVAK

$$\frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2-1} + \frac{E}{s^2+1}$$

$$f(s) = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{1}{2}s+1}{s^2-1} - \frac{\frac{1}{2}}{s^2+1} = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{1}{2}s}{s^2-1} + \frac{1}{s-1} - \frac{\frac{1}{2}}{s^2+1}$$

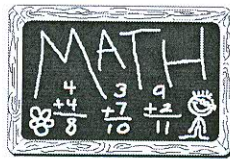
~~F(s) =~~

~~$$F(t) = \frac{1}{2} + \frac{1}{2}t + \frac{1}{2} \cosh(\frac{1}{2}t) + \sinh(\frac{1}{2}t) - \frac{1}{2} \sin(\frac{1}{2}t)$$~~

$$F(t) = \frac{1}{2} + \frac{1}{2}t + \cosh\left(\frac{1}{2}t\right) + \sinh(t) - \sin\left(\frac{1}{2}t\right)$$

UVRSTI U SEDMOŽBE I PROVJERI DA NE ODGOVARA.

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



② $r=1$

$$T\left(\frac{2}{r}, 1\right)$$

$$\iint_K (3-2y) dx dy$$

$$\int_0^{2\pi} \int_0^1 (3-2(r \sin \varphi + 1)) r dr \checkmark$$

$$\int_0^{2\pi} \int_0^1 (3-2r \sin \varphi - 2) r dr$$

$$\int_0^{2\pi} \int_0^1 (1-2r \sin \varphi) r dr$$

$$\int_0^{2\pi} \int_0^1 r dr - 2 \int_0^{2\pi} \int_0^1 r^2 \sin \varphi dr$$

~~$$\int_0^{2\pi} \sin \varphi d\varphi \int_0^1 r dr - 2 \int_0^{2\pi} \int_0^1 r^2 dr$$~~

$$\int_0^{2\pi} \int_0^1 r dr - 2 \sin \varphi \int_0^1 r^2 dr$$

$$\int_0^{2\pi} \left[\frac{r^2}{2} \Big|_0^1 - 2 \sin \varphi \frac{r^3}{3} \Big|_0^1 \right]$$

$$\int_0^{2\pi} \left[\left(\frac{1^2}{2} - \frac{0^2}{2} \right) - \left(2 \sin \varphi \left(\frac{1^3}{3} - \frac{0^3}{3} \right) \right) \right]$$

$$\int_0^{2\pi} \left[\frac{1}{2} - 2 \sin \varphi \frac{1}{3} \right]$$

$$\int_0^{2\pi} \left[\frac{1}{2} - \frac{2}{3} \sin \varphi \right]$$

$$\frac{1}{2} \int_0^{2\pi} d\varphi - \frac{2}{3} \int_0^{2\pi} \sin \varphi d\varphi$$

$$\frac{1}{2} \varphi \Big|_0^{2\pi} + \frac{2}{3} \cos \varphi \Big|_0^{2\pi}$$

$$\left[\left(\frac{1}{2} \cdot 2\pi \right) - \left(\frac{1}{2} \cdot 0 \right) \right] + \left[\left(\frac{2}{3} \cos 2\pi \right) - \left(\frac{2}{3} \cos 0 \right) \right]$$

$$\left[\pi - 0 \right] + \left[\frac{2}{3} - \left(\frac{2}{3} \cdot 1 \right) \right]$$

$$\pi + \left(\frac{2}{3} - \frac{2}{3} \right)$$

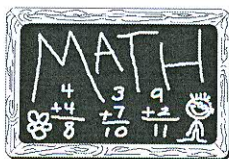
$$\pi + 0 = \pi \checkmark$$

$$x = r \cos \varphi + p \Rightarrow r \cos \varphi d\varphi + 2$$

$$y = r \sin \varphi + q \Rightarrow r \sin \varphi d\varphi + 1$$

20

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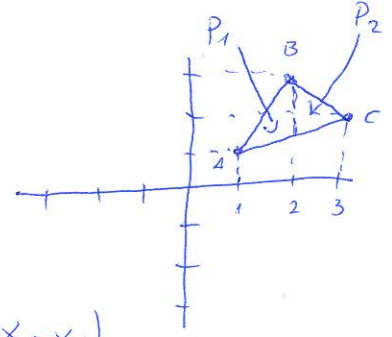


IME I PREZIME: LUKA KURILIĆ

BROJ INDEKSA: 58076

① $A(1,1)$ $B(2,3)$ $C(3,2)$ $f(x,y) = xy$

$$\iint_T f(x,y) dx dy$$



$$AB \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{3 - 1}{2 - 1} (x - 1)$$

$$y - 1 = 2(x - 1)$$

$$y - 1 = 2x - 2$$

$$y = 2x - 2 + 1$$

$$\overline{AB} \Rightarrow \boxed{y = 2x - 1}$$

$$BC \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 3 = \frac{2 - 3}{3 - 2} (x - 2)$$

$$y - 3 = \frac{-1}{1} (x - 2)$$

$$y - 3 = -x + 2$$

$$y = -x + 2 + 3$$

$$\overline{BC} \Rightarrow \boxed{y = -x + 5}$$

$$AC \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{2 - 1}{3 - 1} (x - 1)$$

$$y - 1 = \frac{1}{2} (x - 1)$$

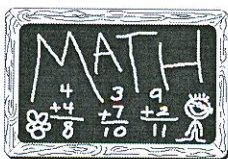
$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x - \frac{1}{2} + 1$$

$$\boxed{y = \frac{1}{2}x - \frac{3}{2}} \Leftarrow \overline{AC}$$

$$\bar{I} = \int_1^2 \int_{\frac{1}{2}x - \frac{3}{2}}^{2x-1} xy dx dy + \int_2^3 \int_{\frac{1}{2}x - \frac{3}{2}}^{-x+5} xy dx dy$$

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POPUNJAVA

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

NASTAVNIK

IME I PREZIME: MATIJA ŠKIBOLA

BROJ INDEKSA:

Broj ↓

bodova

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

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- Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2 + 3y) dx dy$. 20
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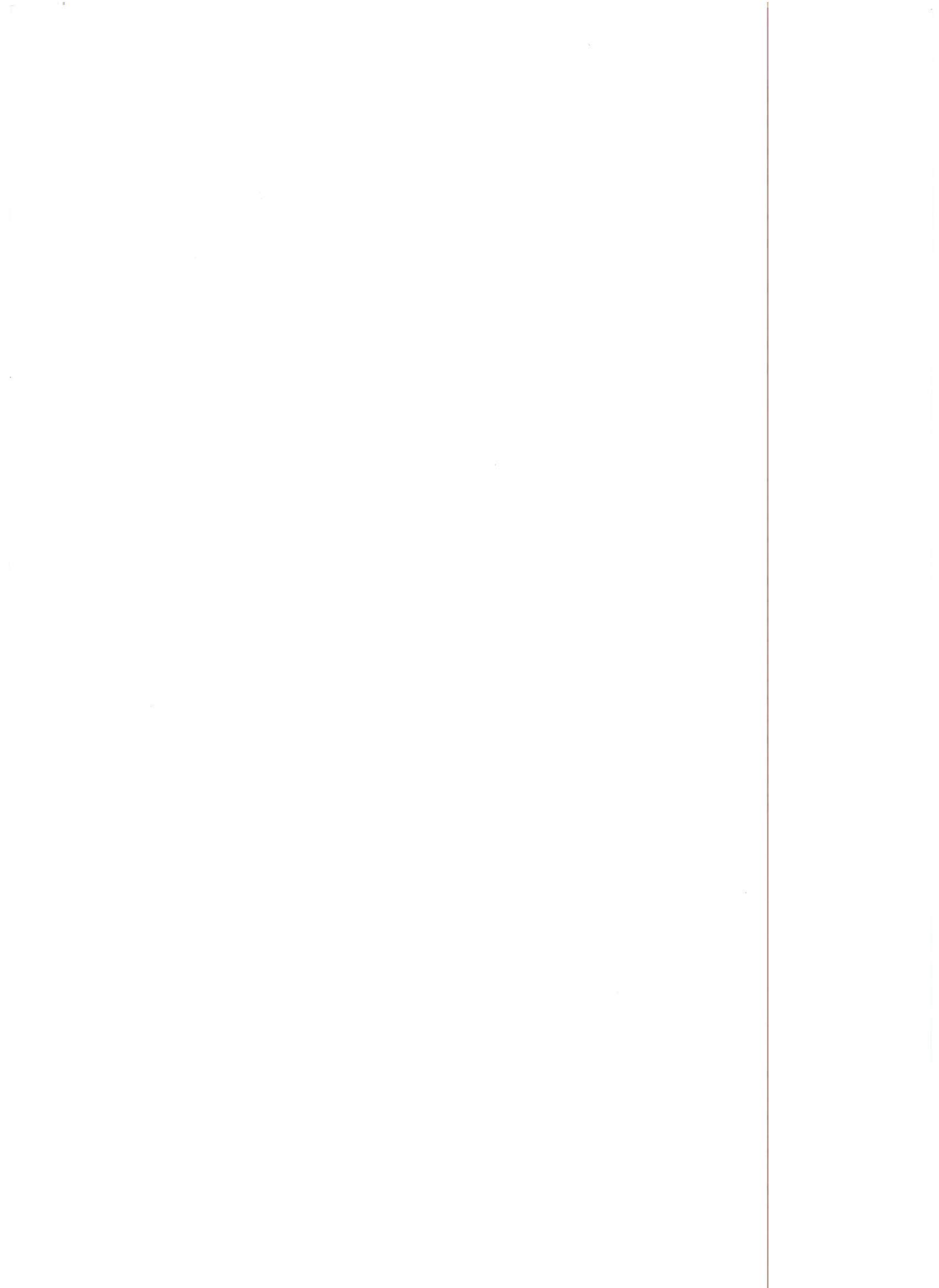
Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

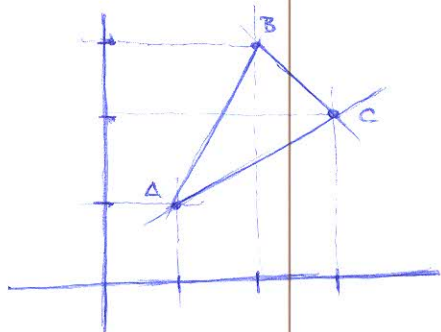
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Laplaceovih transformacija:

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t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



1. $A(x_1, y_1) = A(1, 1)$ $B(x_2, y_2) = B(2, 3)$ $C(x_2, y_2) = C(3, 2)$



$$AB \dots y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$AC \dots y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 1 = \frac{3 - 1}{2 - 1} (x - 1)$$

$$y - 1 = \frac{2 - 1}{3 - 1} (x - 1)$$

$$y - 1 = \frac{2}{1} (x - 1)$$

$$y - 1 = \frac{1}{2} (x - 1)$$

$$y - 1 = 2x - 2$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = 2x - 2 + 1$$

$$y = \frac{1}{2}x - \frac{1}{2} + 1$$

$$\boxed{y = 2x - 1}$$

$$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$$

$\int_{\frac{1}{2}x + \frac{1}{2}}^{2x - 1} dx/dy = \int dx$

$$y/ = \int dx (2x - 1 - \frac{1}{2}x - \frac{1}{2})$$

$$BC \dots y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$y - 3 = \frac{2 - 3}{3 - 2} (x - 2)$$

$$y - 3 = -\frac{1}{1} (x - 2)$$

$$y - 3 = -x + 2$$

$$y = -x + 2 + 3$$

$$\boxed{y = -x + 5}$$

$$= \int_1^2 2x dx - \int_1^2 dx - \int_1^2 \frac{1}{2}x dx - \int_1^2 \frac{1}{2} dx$$

$$= \frac{2x^2}{2} \Big|_1^2 - x \Big|_1^2 - \frac{1}{2} \cdot \frac{x^2}{2} \Big|_1^2 - \frac{1}{2} x \Big|_1^2$$

$$= \frac{2}{2} \cdot (2^2 - 1^2) - 1 - \frac{1}{2} \cdot \frac{1}{2} (2^2 - 1^2) - \frac{1}{2} (2 - 1) = \frac{2}{2} \cdot 5 - 1 - \frac{1}{2} \cdot \frac{1}{2} \cdot 5 - \frac{1}{2} \cdot 1$$

$$= \frac{5}{2} - 1 - \frac{1}{2} \cdot \frac{5}{2} - \frac{1}{2} = \frac{5}{2} - 1 - \frac{5}{4} - \frac{1}{2} = \frac{10 - 4 - 5 - 2}{4} = -\frac{1}{4} \quad \boxed{P = \left| \frac{1}{4} \right|}$$

$\int_{\frac{1}{2}x + \frac{1}{2}}^{-x + 5} dx/dy = \int dx$

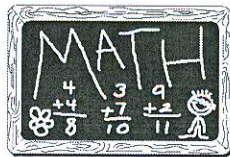
$$y/ = \int dx (-x + 5 - \frac{1}{2}x - \frac{1}{2}) = \int_2^3 -x dx + \int_2^3 5 dx - \int_2^3 \frac{1}{2}x dx - \int_2^3 \frac{1}{2} dx$$

$$= -\frac{x^2}{2} \Big|_2^3 + 5x \Big|_2^3 - \frac{1}{2} \cdot \frac{x^2}{2} \Big|_2^3 - \frac{1}{2} x \Big|_2^3 = -\frac{1}{2} \cdot (3^2 - 2^2) + 5(3 - 2) - \frac{1}{2} \cdot \frac{1}{2} (3^2 - 2^2) - \frac{1}{2} (3 - 2)$$

$$= -\frac{1}{2} \cdot 13 + 5 - \frac{1}{2} \cdot \frac{1}{2} \cdot 13 - \frac{1}{2} \cdot 1 = -\frac{13}{2} + 5 - \frac{1}{2} \cdot \frac{13}{2} - \frac{1}{2} = -\frac{13}{2} + 5 - \frac{13}{4} - \frac{1}{2} = \frac{-26 + 20 - 13 - 2}{4} = -\frac{21}{4} \quad P = \left| \frac{21}{4} \right|$$

$$P = \frac{1}{4} + \frac{21}{4} \quad P = \frac{22}{4} \quad \boxed{P = \frac{11}{2}}$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



5) $f''(t) - f'(t) = \cos(t)$ $f(0) = 1, f'(0) = f''(0) = 0$

$$s^3 f(s) - s^2 \cdot f(0) - s \cdot f'(0) - f''(0) - s f(s) - f(s) = \frac{s}{s^2 + 1}$$

$$s^3 f(s) - s^2 \cdot 1 - s \cdot 0 - 0 - s f(s) - f(s) = \frac{s}{s^2 + 1}$$

~~$f(s)(s^3 - s) = \frac{s}{s^2 + 1} + 1 + s^2$~~

$$s^3 f(s) - s^2 - s f(s) - 1 = \frac{s}{s^2 + 1}$$

$$f(s)(s^3 - s) = \frac{s}{s^2 + 1} + 1 + s^2$$

~~$f(s)(s^3 - s) = s f(s)(s^2 + 1)(1 + s^2) \Rightarrow s + s^3 + s^5 + s^7$~~

$$f(s) = \frac{s + 1 + s^2}{s^2 + 1} \Rightarrow \frac{s + 1 + s^2}{(s^2 + 1)(s^3 - s)} \Rightarrow \frac{s + 1 + s^2}{(s^2 + 1)s(s^2 - 1)} \quad \frac{s^2 + s + 1}{(s^2 + 1)s(s^2 - 1)}$$

$$f(s) = \frac{As + B}{s^2 + 1} + \frac{C}{s} + \frac{Ds + E}{s^2 - 1}$$

$$f(s) = (As + B)s(s^2 - 1) + C(s^2 + 1)(s^2 - 1) + (Ds + E)(s^2 + 1)s$$

$$f(s) = (As^2 + Bs)(s^2 - 1) + (Cs^2 + C)(s^2 - 1) + (Ds^2 + Es)(s^2 + 1)$$

$$= AS^4 - AS^2 + BS^3 - BS + CS^4 - CS^2 + CS^2 - C + Ds^3 + Ds + Es^3 + Es$$

$$f(s) = s^4(A + C + D) + s^3(B + E) + s^2(A + D) + s(B + E) + C$$

~~$A + C + D = 0$~~

~~$B + E = 0$~~

~~$-A + D = 1$~~

~~$-B + E = 1$~~

$-C = 1 \Rightarrow C = -1$



ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.

