

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj

odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!

IME I PREZIME: DINO KURIC

BROJ INDEKSA: 56192-2008

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

POPUNJAVA

NASTAVNIK

Broj ↓

bodova

- Zadan trokut T sa vrhovima: $A(1, 1)$, $B(2, 3)$ i $C(3, 2)$ i funkcija $f(x, y) = xy$. Odrediti $\iint_T f(x, y) dx dy$. 20
- Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2, 1)$. Izračunati $\iint_K (3 - 2y) dx dy$. 20
- Provjeriti da li je krivoljni integral u vektorskom polju $g(x, y, z) = (2x - 1, 3y + z, 2z + y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
- Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2 + 3y) dx dy$. 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0.$$

Tablica integrala

Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left x + \sqrt{x^2 \pm a^2} \right + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

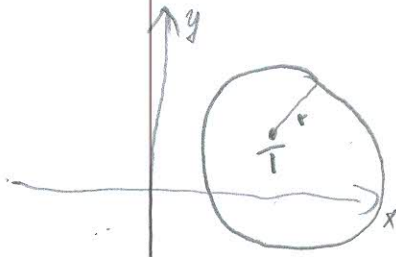
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Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



2. $r=1$ $T(2,1)$; $\iint_K (3-2y) dx dy =$



Polarne koordinate

$$x = r \cos \varphi + 2$$

$$y = r \sin \varphi + 1$$

$$r \in (0, 1)$$

$$\varphi \in (0, 2\pi)$$

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2r \sin \varphi) r dr$$

$$\int_0^{2\pi} d\varphi \cdot 4 \int_0^1 -2r \sin \varphi dr$$

$$\int_0^{2\pi} \sin \varphi d\varphi \cdot 4 \int_0^1 -2r \cdot r dr$$

$$4 \int_0^{2\pi} -\cos \varphi d\varphi \cdot 4 \cdot (-2) \cdot \int_0^1 r^2 dr$$

$$= -\cos \varphi \Big|_0^{2\pi} \cdot 8 \cdot \frac{r^3}{3} \Big|_0^1$$

0,33

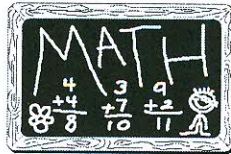
$$= -\cos 2\pi \Big|_0^{2\pi} - 8 \cdot \frac{1}{3} \Big|_0^1$$

$$= -(\cos 2\pi + \cos 0) \Big|_0^{2\pi} - 8 \cdot \frac{1}{3}$$

$$= -1 + 1 -$$

$$= -0,33$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



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