

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod!!

IME I PREZIME: **MATE BALJAK**

BROJ INDEKSA: **57115**

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

POPUNJAVA  
NASTAVNIK  
Broj ↓  
bodova

- Zadan trokut  $T$  sa vrhovima:  $A(1, 1)$ ,  $B(2, 3)$  i  $C(3, 2)$  i funkcija  $f(x, y) = xy$ . Odrediti  $\iint_T f(x, y) dx dy$ . 20
- Neka je  $K$  krug radijusa  $r = 1$  sa centrom u točki  $T(2, 1)$ . Izračunati  $\iint_K (3 - 2y) dx dy$ . 20
- Provjeriti da li je krivuljni integral u vektorskom polju  $g(x, y, z) = (2x - 1, 3y + z, 2z + y)$  neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
- Neka je  $K$  kocka stranice duljine  $a = 2$  centrirana u ishodištu. Izračunati  $\iint_{\partial K} (2 + 3y) dx dy$ . 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, f'(0) = f''(0) = 0.$$

Tablica integrala

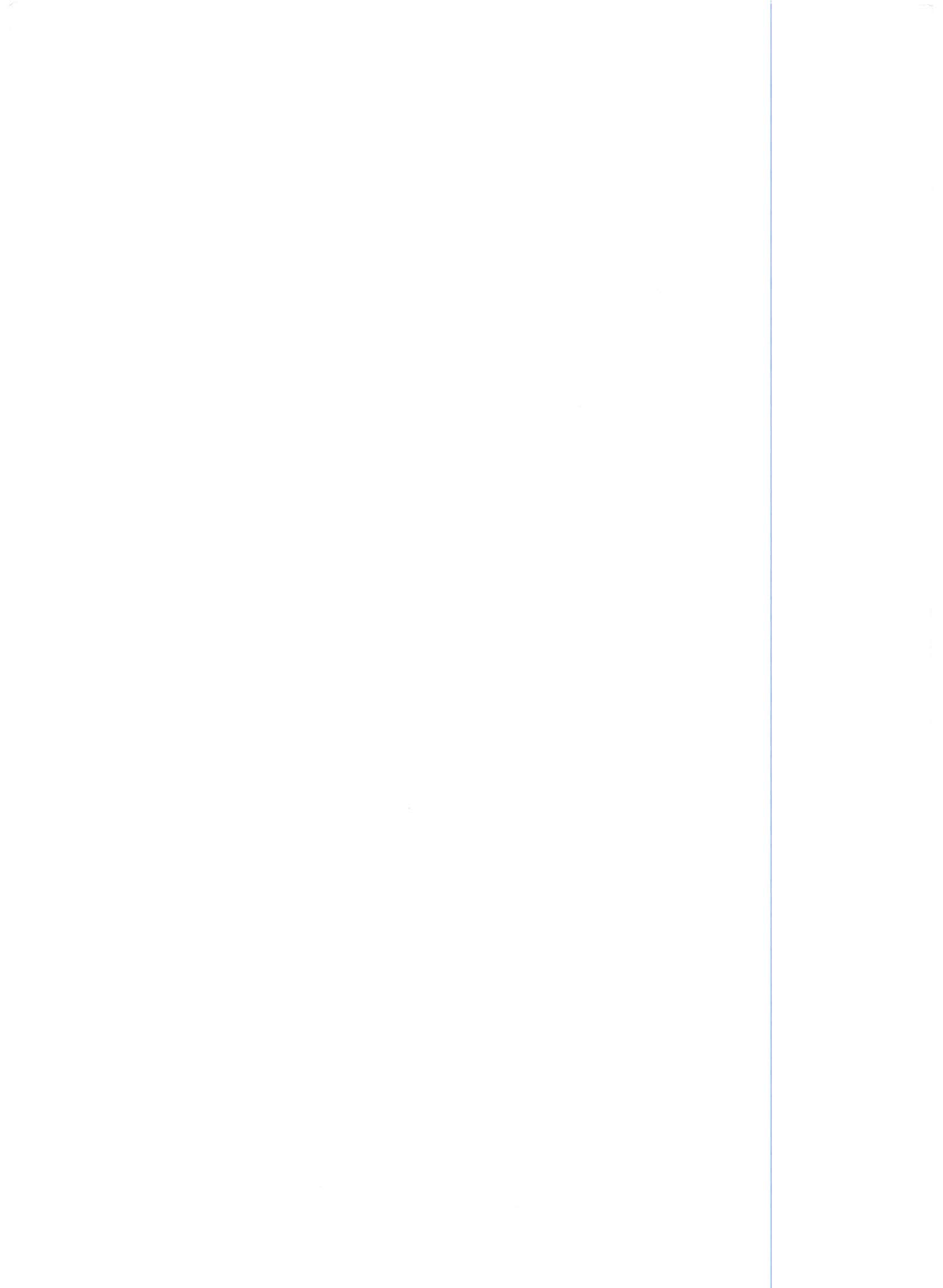
Ukupno: 20

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Tablica

Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$c$	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$t$	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



5)  $f'''(x) - f'(x) = \cos(x)$ ,  $f(0) = 1$ ,  $f'(0) = f''(0) = 0$

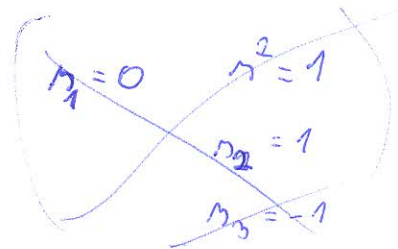
$$n^3 F(n) - n^2 f'(0) - n f''(0) - f'''(0) - n F(n) + f(0) = \frac{n}{n^2 + 1}$$

$$n^3 F(n) - n^2 - n F(n) + 1 = \frac{n}{n^2 + 1}$$

$$F(n) n(n^2 - 1) = \frac{n}{n^2 + 1} + n^2 - 1$$

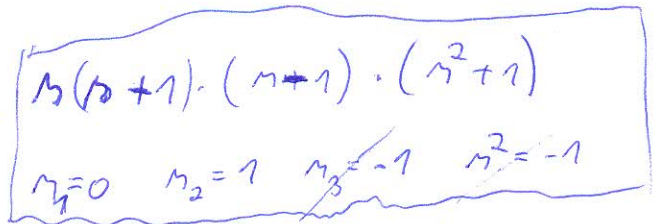
$$F(n) n(n^2 - 1) = \frac{n + n^4 - n^2 + n^2 - 1}{n^2 + 1}$$

$$F(n) n(n^2 - 1) = \frac{n^4 + n - 1}{n^2 + 1} \cdot \frac{1}{n(n^2 - 1)}$$



~~$F(n) = \frac{n^4 + n - 1}{n(n^2 - 1)}$~~

$$F(n) = \frac{n^4 + n - 1}{n(n^2 - 1) \cdot (n^2 + 1)} = \frac{n^4 + n - 1}{n(n+1) \cdot (n-1) \cdot (n^2 + 1)}$$



$$= \frac{A}{n} + \frac{B}{n-1} + \frac{C+D+E}{(n+1)(n^2+1)} = \frac{A}{n} + \frac{B}{n-1} + \frac{C}{n+1} + \frac{D+E}{n^2+1}$$

$$= A(n^5 - n^4 + n^3 - n^2 + n - 1) + n^5 + n^4 + n^3 + n^2 - n - 1 + n^6 + n^4 + n^3 + n^2 - n - 1$$

~~$$= A(n^6 + 2n^5 + n^4 + n^3 + n^2 - n - 1)$$~~

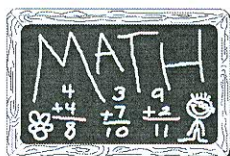
~~$$B(n^5 + n^2 - n + n^5 + n^4)$$~~

~~AB~~

~~$$n^4 + n - 1 = A(n^4 + n^2 + n^3 + n^2 - n - 1) + B(n^4 + n^2 + n^3 + n)$$~~

$$n^4 + n - 1 = A(n^4 + n^2 - n^3 - n + n^3 + n - n^2 - 1) + B(n^4 + n^2 - n^3 - n) + C(n^4 + n^2 + n^3 + n) + D + E(n^3 - n^2 + n^2 - n)$$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



$m=0$

$m=-1$

$m=1$

$-1 = -A$

$-1 = 4B$

$1 = 4C$

$A=1$

$B = -\frac{1}{4}$

$C = \frac{1}{4}$

$\frac{1}{m^4 + m + 1}$

$m^4 + m - 1 = Am^4 - A + Bm^4 - Bm^3 + Bm^2 - Bm + Cm^4 + Cm^3 + Cm^2 + Cm + Dm^4 - Dm^2 + Em^3 - Em$

$m^4 : 1 = A + B + C + D$

$D = 1 - A - B - C$

$D = 1 - 1 + \frac{1}{4} - \frac{1}{4} = 0$

$m^3 : 0 = -B + C + E \rightarrow E = B - C$

$E = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$F(m) = \frac{1}{m} - \frac{1}{4} \cdot \frac{1}{m+1} + \frac{1}{4} \cdot \frac{1}{m-1} - \frac{1}{2} \cdot \frac{1}{m^2+1}$

$\frac{-\frac{1}{4}}{s+1} + \frac{\frac{1}{4}}{s-1} =$

~~$f(t) = 1 - \frac{1}{4}e^{-t} + \frac{1}{4}e^t - \frac{1}{2}\sin(t)$~~

~~$f(t) = 1 + \frac{1}{4} \frac{1}{m+1} + \frac{1}{4} \frac{1}{m-1} - \frac{1}{2} \frac{1}{m^2+1}$~~

$\frac{\frac{2}{4}}{m^2-1} = \frac{1}{2} \frac{1}{m^2-1}$

~~$f(t) = 1$~~

~~$f(t) = 1$~~

$= 1 - \frac{1}{16} \sin \ln(t) - \frac{1}{2} \sin(t)$  X

X

