MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇊

Ime i prezime: MATE BALJAK **Broj indeksa:** 57115

Vrijeme početka: **Vrijeme završetka:**

Popunjava nastavnik
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = xy$. Odrediti $\iint_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x-1,\, 3y+z,\, 2z+y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2+3y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

IME I PREZIME: MATE BALJAK   BROJ INDEKSA: 57115

5. $f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0$

$$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) - sF(s) + f(0) = \frac{s}{s^2+1}$$

$$s^3F(s) - s^2 - sF(s) + 1 = \frac{s}{s^2+1}$$

$$F(s)\, s(s^2-1) = \frac{s}{s^2+1} + s^2 - 1$$

$$F(s)\, s(s^2-1) = \frac{s + s^4 \cancel{- s^2} \cancel{+ s^2} - 1}{s^2+1}$$

$$F(s)\, s(s^2-1) = \frac{s^4 + s - 1}{s^2+1} \quad \Big| \cdot \frac{1}{s(s^2-1)}$$

$\cancel{s_1 = 0 \quad s^2 = 1 \quad s_2 = 1 \quad s_3 = -1}$

$\cancel{F(s) = s^4 + s}$

$$F(s) = \frac{s^4+s-1}{s(s^2-1)\cdot(s^2+1)} = \frac{s^4+s-1}{s(s+1)\cdot(s-1)\cdot(s^2+1)}$$

$$\boxed{\begin{array}{l} s(s+1)\cdot(s-1)\cdot(s^2+1) \\ s_1 = 0 \quad s_2 = 1 \quad s_3 = -1 \quad s^2 = -1 \end{array}}$$

$$= \cancel{\frac{A}{s} + \frac{B}{s-1} + \frac{C+Ds}{(s+1)\cdot(s^2+1)}} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1} + \frac{D+Es}{s^2+1} \quad \Big| \cdot s(s+1)(s-1)\cdot(s^2+1)$$

$$\cancel{= A(s^5 - s^4 + s^2 - s - s + 1 + s^5 + s^4 + s^2 + s - s - 1 + s^6 + s^4 + s^3 + s - s^2 - 1)}$$

$$\cancel{= A(s^6 + 2s^5 + s^4 + s^3 + s^2 - s - 1)}$$

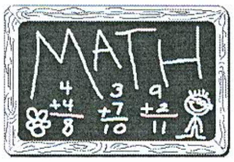
$$\cancel{B(s^5 + s^2 - s + s^5 + s^4}$$

$$\cancel{s^4 + s + 1 = A(s^4 + s^2 + s^2 + s - s^3 - s - s^2 - 1) + B(s^4 + s^2 + s^3 + s) +}$$

C

$$s^4 + s - 1 = A(s^4 + \cancel{s^2} - \cancel{s^3} - \cancel{s} + \cancel{s^3} + \cancel{s} - \cancel{s^2} - 1) + B(s^4 + s^2 - s^3 - s) + C(s^4 + s^2 + s^3 + s)$$
$$+ D + Es(s^3 - \cancel{s^2} + \cancel{s^2} - s)$$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



IME I PREZIME: MATE BALJAK

BROJ INDEKSA: 57115

za $s=0$

$-1=-A$

$A=1$

za $s=-1$

$-1=4B$

$B=-\frac{1}{4}$

za $s=1$

$1=4C$

$C=\frac{1}{4}$

~~s^4+s+1~~

$s^4+s-1 = As^4 - A + Bs^4 - Bs^3 + Bs^2 - Bs + Cs^4 + Cs^3 + Cs^2 + Cs + Ds^4 - Ds^2 + Es^3 - Es$

uz s^4 : $1 = A+B+C+D$

$D = 1-A-B-C$

$D = 1-1+\frac{1}{4}-\frac{1}{4} = \boxed{0}$

uz s^3 : $0 = -B+C+E \rightarrow E = B-C$

$E = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$F(s) = \frac{1}{s} - \frac{1}{4}\cdot\frac{1}{s+1} + \frac{1}{4}\cdot\frac{1}{s-1} - \frac{1}{2}\cdot\frac{1}{s^2+1}$

$\frac{-\frac{1}{4}}{s+1} + \frac{\frac{1}{4}}{s-1} =$

~~$f(t) = 1 - \frac{1}{4}e^{-t} + \frac{1}{4}e^{t} - \frac{1}{2}\sin(t)$~~

~~$\frac{\frac{2}{4}}{s^2-1^2} = \frac{1}{2}\frac{1}{s^2-1^2}$~~

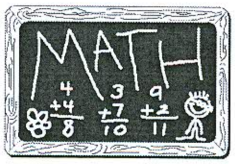
~~$f(t) = 1 + \frac{\frac{1}{4}}{s+1} + \frac{\frac{1}{4}}{s-1} - \frac{1}{2}\frac{1}{s^2+1}$~~

~~$f(t) = 1 -$~~

$f(t) = 1$ ~~$\frac{1}{4}$~~

$= 1 - \frac{1}{16}\sinh(t) - \frac{1}{2}\sin(t)$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: TONI ŠESTAN **BROJ INDEKSA:** 55283-2007

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y)=xy$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. ~~20~~
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5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t)-f'(t)=\cos(t), \quad f(0)=1,\ f'(0)=f''(0)=0.$$

Ukupno: 35

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica

Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

5. $f'''(t) - f'(t) = \cos(t) \qquad f(0) = 1, \quad f'(0) = f''(0) = 0$

$$s^3F(s) - s^2f(0) - s\cancel{f'(0)}^{0} - \cancel{f''(0)}^{0} - (sF(s) - f(0)) = \frac{s}{s^2+1}$$

$$s^3F(s) - s^2 - sF(s) + 1 = \frac{s}{s^2+1}$$

$$s^3F(s) - sF(s) = \frac{s}{s^2+1} + \frac{s^2}{1} - 1$$

$$F(s)(s^3 - s) = \frac{s + s^2(s^2+1) - 1(s^2+1)}{s^2+1}$$

$$F(s)(s^3 - s) = \frac{s + s^4 + \cancel{s^2} - \cancel{s^2} - 1}{s^2+1}$$

$$F(s) = \frac{s^4 + s - 1}{s(s^2-1)(s^2+1)}$$

$$F(s) = \frac{s^4 + s - 1}{s(s-1)(s+1)(s^2+1)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+1} + \frac{Ds+E}{s^2+1}$$

$$s^4 + s - 1 = A(s^4 - 1) + B\underbrace{(s^2+s)(s^2+1)}_{s^4+s^2+s^3+s} + C\underbrace{(s^2-s)(s^2+1)}_{s^4+s^2-s^3-s} + (Ds+E)(s^3-s)$$

$$s^4 + s - 1 = As^4 - A + Bs^4 + Bs^2 + Bs^3 + Bs + Cs^4 + Cs^2 - Cs^3 - Cs + Ds^4 - Ds^2 + Es^3 - Es$$

$1 = A + B + C + D \Rightarrow 0 = B + C + \cancel{D}$
$0 = B + C - \cancel{D}$ +

$0 = 2B + 2C$
$1 = 2B - 2C$

$1 = 4B$

$\boxed{B = \frac{1}{4}}$

$\boxed{C = \frac{1}{4}}$

$0 = B - C + E$
$0 = B - D + C$
$1 = B - C - E$
$-1 = -A \Rightarrow \boxed{A = 1}$

$0 = B - C + \cancel{E}$
$1 = B - C - \cancel{E}$ +

$1 = 2B - 2C$

$D = B - C$
$D = -\frac{1}{4} + \frac{1}{4}$
$\boxed{D = 0}$

$E = B + C$
$E = -\frac{1}{4} - \frac{1}{4}$
$\boxed{E = -\frac{2}{4} = -\frac{1}{2}}$

$$F(s) = \frac{1}{s} + \frac{1}{4}\frac{1}{s-1} - \frac{1}{4}\cdot\frac{1}{s+1} - \frac{1}{2}\cdot\frac{1}{s^2+1}$$

$$f(t) = 1 + \frac{1}{4}e^t - \frac{1}{4}e^{-t} - \frac{1}{2}\sin t \quad \checkmark \ 20$$

$f'(t) = \frac{1}{4}e^t + \frac{1}{4}e^{-t} - \frac{1}{2}\cos t$
$f''(t) = \frac{1}{4}e^t - \frac{1}{4}e^{-t} + \frac{1}{2}\sin t$
$f'''(t) = \frac{1}{4}e^t + \frac{1}{4}e^{-t} + \frac{1}{2}\cos t$

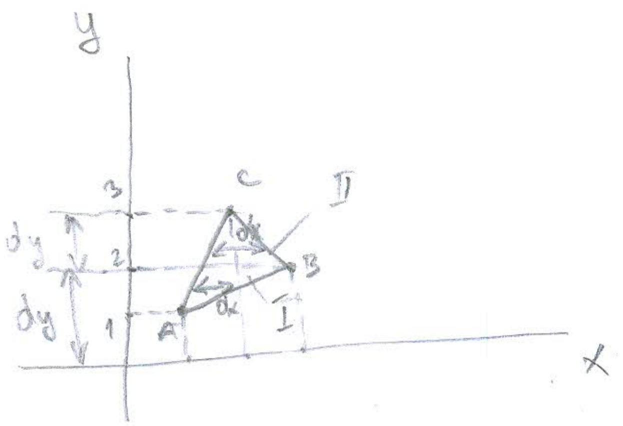
IME I PREZIME: TONI ŠESTAN BROJ INDEKSA: 55283-2007

1.

A (1,1)
B (2,3)
C (3,2)

$f(x,y) = xy$

$\iint_T f(x,y)\,dx\,dy$



I: $\int_1^2 xy\,dx\,dy \int_{-y+2}^{-y+2} xy\,dx\,dy$ X

II: $\int_2^3 xy\,dx\,dy \int_{-y+5}^{-y+2} xy\,dx\,dy$ X

AB: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{3-1}{2-1}(x-1)$

$y - 1 = 2x - 2 \quad / \cdot -\frac{1}{2}$

$y - 1 = -x + 1$

$x = -y + 2$

BC:

$y - 3 = \frac{2-3}{3-2}(x - 2)$

$y - 3 = -x + 2$

$x = -y + 5$

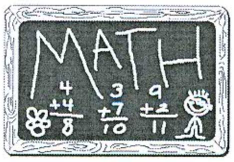
AC: $y - 1 = \frac{2-1}{3-1}(x-1)$

$y - 1 = \frac{1}{2}x - \frac{1}{2} \quad / \left(-\frac{2}{1}\right)$

$y - 1 = -x + 1$

$x = -y + 2$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



Ime i prezime: TONI ŠESTAN

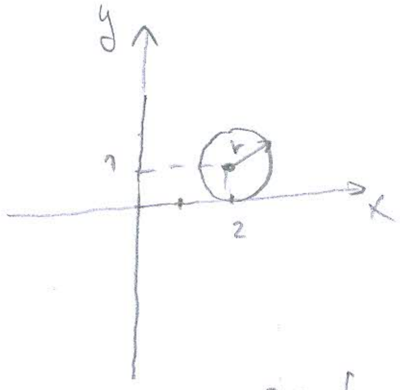
Broj indeksa: 55283-2007

2.

$r=1$

$T(2,1)$

$$\iint_K (3-2y)\,dx\,dy$$



$x = r\cos\varphi + 2$

$y = r\sin\varphi + 1$

$dx\,dy = r\,dr\,d\varphi$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$$\int_0^{2\pi}\int_0^1 \left(3 - 2(r\sin\varphi + 1)\right) r\,dr\,d\varphi \quad \checkmark \; \underline{15}$$

$$\int_0^{2\pi}\int_0^1 (3 - 2r\sin\varphi + 2)\, r\,dr\,d\varphi$$

$$\underbrace{2\int_0^{2\pi}\int_0^1 (r^2\sin\, dr\,d\varphi}_{\text{I.}} - \underbrace{1\iint r\,dr\,d\varphi}_{\text{II.}}$$

$$\text{I}: \quad 2\int_0^{2\pi} \sin\varphi\,d\varphi \int_0^1 r^2\,dr = ?$$

$$\text{II}: \quad 1\int_0^{2\pi} d\varphi \int_0^1 r\,dr = ?$$

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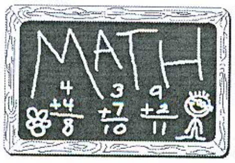

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: ANTE GRUBIŠA **BROJ INDEKSA:** 57831

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y)=xy$. Odrediti $\iint\limits_T f(x,y)\,\mathrm{d}x\mathrm{d}y$. 20 15
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3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z)=(2x-1,\,3y+z,\,2z+y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2+3y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t)-f'(t)=\cos(t)\,,\quad f(0)=1,\ f'(0)=f''(0)=0.$$

Ukupno: 50

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

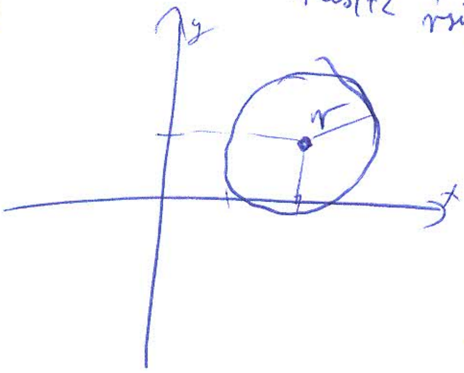
IME I PREZIME: ANTE GRUBIŠA   BROJ INDEKSA: 57831

2. $r = 1$ $T(2,1)$ $\iint_K (3-2y)\,dx\,dy$

$r\cos\varphi + 2$ $r\sin\varphi + 1$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$

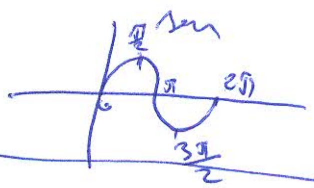


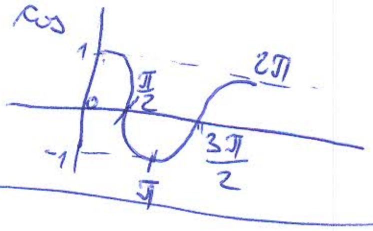
15

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - (2 r\sin\varphi + 2))\, r\,dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2 r\sin\varphi + 2)\, r\,dr = \int_0^{2\pi} d\varphi \ldots = \int_0^{2\pi} d\varphi\; 3\cdot(1-0) = 3\int_0^{2\pi} d\varphi = 3\cdot(2\pi - 0)$$

$$= 6\pi$$





4. KOCKA $a = 2$ $T(0,0)$

$$\iint_{\partial K} (2+3y)\,dx\,dy$$

$$\omega = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2+3y \end{pmatrix}$$

$$\operatorname{div}\omega = \partial_x \omega_x + \partial_y \omega_y + \partial_z \omega_z$$

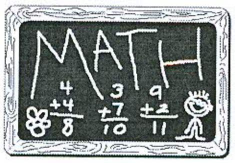
$$= 0 + 0 + 0$$

$$\operatorname{div}\omega = 0$$

$$\iint_{\partial K} (2+3y)\,dx\,dy = \iiint_K \underbrace{0}_{\operatorname{div}\omega}\,dx\,dy\,dz = 0$$

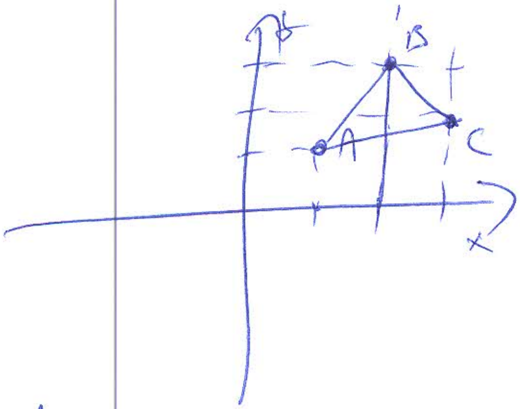
20

Zadatke rješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



IME I PREZIME: ANTE GRUBIŠA BROJ INDEKSA: 57831

1. $A(1,1)$ $B(2,3)$ $C(3,2)$ $f(x,y) = xy$



$$\iint_T f(x,y)\,dx\,dy$$

AB $y-1 = \frac{3-1}{2-1}(x-1)$

15 $y-1 = 2x-2$ $\boxed{y = 2x-1}$

AC $y-1 = \frac{2-1}{3-1}(x-1)$

$y = \frac{1}{2}x - \frac{1}{2} + 1$

$\boxed{y = \frac{1}{2}x + \frac{1}{2}}$

BC $y-3 = \frac{2-3}{3-2}(x-2)$

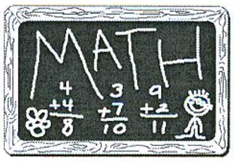
$y = -x+2+3$

$\boxed{y = -x+5}$

$$\int_1^2 \int_{\frac{1}{2}x+\frac{1}{2}}^{2x-1} xy\,dy\,dx + \int_2^3 \int_{\frac{1}{2}x+\frac{1}{2}}^{-x+5} xy\,dy\,dx$$

$= \ldots ?$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

POPUNJAVA NASTAVNIK

IME I PREZIME: BORIS DURBIĆ

BROJ INDEKSA: 57640

VRIJEME POČETKA:

VRIJEME ZAVRŠETKA:

Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y)=xy$. Odrediti $\iint\limits_T f(x,y)\,dx\,dy$. 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,dx\,dy$. 20 15
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z)=(2x-1,\,3y+z,\,2z+y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2+3y)\,dx\,dy$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t)-f'(t)=\cos(t)\,,\quad f(0)=1,\ f'(0)=f''(0)=0.$$

Ukupno: 35

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
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$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
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t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

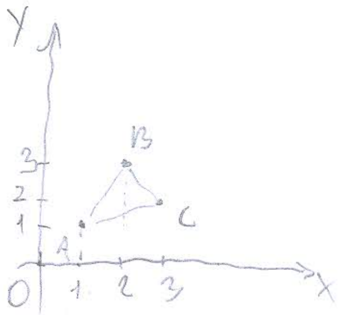
Ime i prezime: BORIS DURBIĆ

Broj indeksa:

1. $A(1,1)$ $\quad f(x,y)=xy$

$B(2,3)$

$C(3,2)$ $\quad \iint_T f(x,y)\,dx\,dy$



BC

$$y-3=\frac{2-3}{3-2}(x-2)$$

$$y-3=-\frac{1}{1}(x-2)$$

$$y-3=-x+2$$

$$y=-x-1$$

AC

$$y-1=\frac{2-1}{3-1}(x-1)$$

$$y-1=\frac{1}{2}x-\frac{1}{2}$$

$$y=\frac{1}{2}x+\frac{1}{2}$$

AB... $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y-1=\frac{3-1}{2-1}(x-1)$$

$$y-1=2(x-1)$$

$$y=2x-2+1$$

$$y=2x-1$$

$$\int_1^2\int_{\frac{1}{2}x+\frac{1}{2}}^{2x-1} xy\,dy\,dx+\int_2^3\int_{\frac{1}{2}x+\frac{1}{2}}^{-x-1} xy\,dy\,dx=$$

0

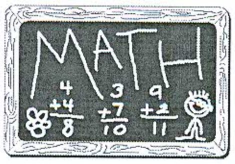
$$=\int_1^2\left(x\left(\frac{y^2}{2}\Big]_{\frac{1}{2}x+\frac{1}{2}}^{2x-1}\right)\right)dx+\int_2^3\left(x\left(\frac{y^2}{2}\Big]_{\frac{1}{2}x+\frac{1}{2}}^{-x+5}\right)\right)dx$$

$$=\int_1^2 x\left(\frac{(2x-1)^2}{2}-\frac{(\frac{1}{2}x+\frac{1}{2})^2}{2}\right)dx+\int_2^3 x\left(\frac{(5-x)^2}{2}-\frac{(\frac{1}{2}x+\frac{1}{2})^2}{2}\right)dy$$

$$=\int_1^2 x\left(\frac{4x^2-4x+1}{2}-\frac{\frac{1}{4}x^2+\frac{1}{4}x+\frac{1}{4}}{2}\right)dx+\int_2^3 x\left(\frac{25-10x+x^2}{2}-\frac{\frac{1}{4}x^2+\frac{1}{4}x+\frac{1}{4}}{2}\right)dx$$

$$\int_1^2 2x^3-2x^2+\frac{1}{2}x-\frac{1}{8}x^3+\frac{1}{8}x^2+\frac{1}{8}x\,dx+\int_2^3\frac{25}{2}x-5x^2+\frac{x^3}{2}-\frac{1}{8}x^3-\frac{1}{8}x^2-\frac{1}{8}x$$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



IME I PREZIME: BORIS ĐURBIĆ

BROJ INDEKSA:

4. $\iint_{\partial K} (2+3y)\,dx\,dy$ $\quad v = \begin{bmatrix} 0 \\ 0 \\ 2+3y \end{bmatrix}$ $\operatorname{div} v = 0$

$$\iint_{\partial K} (3-2y)\,dx\,dy = \iiint_K \operatorname{div} v = 0$$

20

2. $r=1$ $T(2,1)$ $\iint_K (3-2y)\,dx\,dy$

$$x = r\cos\varphi + 2$$
$$y = r\sin\varphi + 1$$

$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2(r\sin\varphi + 1))\cdot r\,dr = \int_0^{2\pi} d\varphi \int_0^1 (3r - 2r^2\sin\varphi)\,dr$$

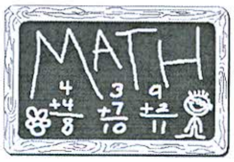
$$= \int_0^{2\pi} d\varphi\, 3\frac{r^2}{2}\Big]_0^1 - \int_0^{2\pi} d\varphi\, 2\frac{r^3}{3}\sin\varphi\Big]_0^1$$

15

$$= \frac{3}{2}\int_0^{2\pi} d\varphi - \frac{2}{3}\int_0^{2\pi} \sin\varphi\, d\varphi$$

$$= \frac{3}{2}\cdot 2\pi - \frac{2}{3}(-\cos\varphi\Big]_0^{2\pi} = 3\pi + \frac{2}{3}\cancel{\cos 2\pi}^{1} - \frac{2}{3}\cancel{\cos 2\pi}^{1} = 3\pi$$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!** Obavezno popuniti sva polja ispod⇓⇓	Popunjava nastavnik Broj ↓ bodova
Ime i prezime: VEDRAN DELAŠ **Broj indeksa:** 52706	
Vrijeme početka: **Vrijeme završetka:**	

1. Zadan trokut T sa vrhovima: $A(1,1)$, $B(2,3)$ i $C(3,2)$ i funkcija $f(x,y) = xy$. Odrediti $\iint\limits_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20 15
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(2,1)$. Izračunati $\iint\limits_K (3-2y)\,\mathrm{d}x\mathrm{d}y$. 20
3. Provjeriti da li je krivuljni integral u vektorskom polju $g(x,y,z) = (2x-1,\, 3y+z,\, 2z+y)$ neovisan o putu, odnosno da li zavisi samo od početne i završne točke? 20
4. Neka je K kocka stranice duljine $a=2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2+3y)\,\mathrm{d}x\mathrm{d}y$. 20
5. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Ime i prezime: VEDRAN DELAŠ Broj indeksa: 52706

2. $r = 1\,\mathrm{cm}$

$T(2,1)$

$$\iint_K (3-2y)\,dx\,dy$$

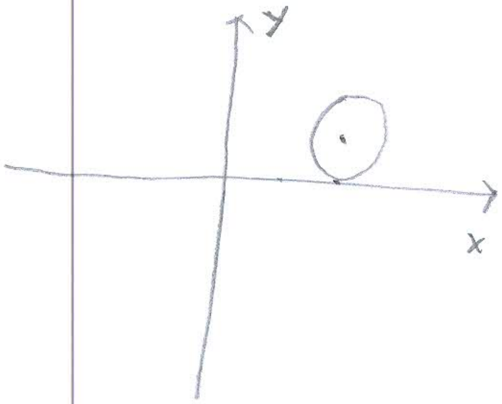
$x = r\cos\varphi$

$y = r\sin\varphi$

~~$\varphi \in 2\pi$~~

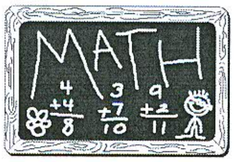
$\varphi \in [0, 2\pi]$

$r \in [0,1]$



$$\int_0^{2\pi} d\varphi \int_0^1 (3 - 2r\sin\varphi)\, r\,dr$$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.

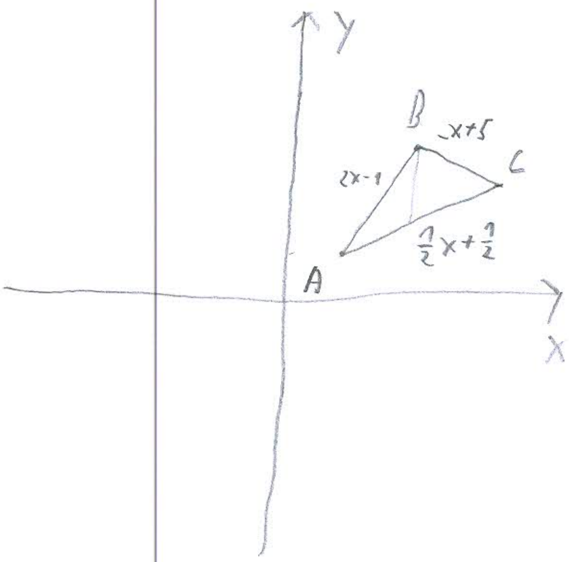


IME I PREZIME: VEDRAN DELAŠ   BROJ INDEKSA: 52206

1. $A(1,1)$ $B(2,3)$ $C(3,2)$

$f(x,y) = xy$

$$\iint_T f(x,y)\,dx\,dy$$



AB $A(\overset{x_1}{1},\overset{y_1}{1}), B(\overset{x_2}{2},\overset{y_2}{3})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{3-1}{2-1}(x-1)$$

$$y - 1 = \frac{2}{1}(x-1)$$

$$y - 1 = 2x - 2$$

$$y = 2x - 1$$

AC $A(\overset{x_1}{1},\overset{y_1}{1}), C(\overset{x_2}{3},\overset{y_2}{2})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{2-1}{3-1}(x-1)$$

$$y - 1 = \frac{1}{2}(x-1)$$

$$y - 1 = \frac{1}{2}x - \frac{1}{2}$$

$$y = \frac{1}{2}x + \frac{1}{2}$$

~~BC~~ BC $B(\overset{x_1}{2},\overset{y_1}{3}), C(\overset{x_2}{3},\overset{y_2}{2})$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 3 = \frac{2-3}{3-2}(x-2)$$

$$y - 3 = \frac{-1}{1}(x-2)$$

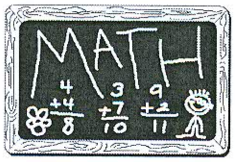
$$y - 3 = -x + 2$$

$$y = -x + 5$$

15

$$\int_1^2 \int_{\frac{1}{2}x+\frac{1}{2}}^{2x-1} (xy)\,dy\,dx + \int_2^3 \int_{\frac{1}{2}x+\frac{1}{2}}^{-x+5} (xy)\,dy\,dx = \ldots ?$$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: Portada Jure **BROJ INDEKSA:** 57350

VRIJEME POČETKA: 14:00 **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y) = \mathrm{e}^{xy}$. Odrediti $\iint\limits_T f(x,y)\,\mathrm{d}x\mathrm{d}y$. 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$. Izračunati iz definicije $\int\limits_{\partial K} (3-2y)\,\mathrm{d}s$. 20
3. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$, a $\overset{\frown}{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\overset{\frown}{\partial K}} (x-y)\,\mathrm{d}y$. 20
4. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 0,\ x'(0) = 5.$$
5. Plohama $x=0$, $y=0$, $z=0$ i $x+y+z=-1$ omeđena je piramida P. Plašt piramide usmjeren prema van označen je sa ∂P. Izračunati $\iint\limits_{\partial P} (z-y)\,\mathrm{d}y\mathrm{d}z$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$A(-1,-1)$ $\quad f(x,y)=e^{xy}$ $\quad$ Odredi $\iint\limits_{T} f(x,y)\,dx\,dy$

$B(0,3)$

$C(2,2)$