MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

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VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y)=\mathrm{e}^{xy}$. Odrediti $\iint_T f(x,y)\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$. Izračunati iz definicije $\int_{\partial K}(3-2y)\,\mathrm{d}s$. 20

3. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$, a $\overset{\frown}{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\overset{\frown}{\partial K}}(x-y)\,\mathrm{d}y$. 20

4. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t)+x'(t)=0,\quad x(0)=x''(0)=0,\ x'(0)=5.$$

5. Plohama $x=0$, $y=0$, $z=0$ i $x+y+z=-1$ omeđena je piramida P. Plašt piramide usmjeren prema van označen je sa ∂P. Izračunati $\iint_{\partial P}(z-y)\,\mathrm{d}y\mathrm{d}z$. 20

Ukupno: 20

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

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$x'''(t) + x'(t) = 0$

$x(0) = 0$

$x''(0) = 0$

$x'(0) = 5$

$s^3 X(s) - s^2 x(0) - s\, x'(0) - x''(0) + s X(s) - x(0) = 0$

$s^3 X(s) + 5s + s X(s) = 0$

$s^3 X(s) + s X(s) = 5s$

$X(s)(s^3 + s) = 5s$

$X(s) = \frac{5s}{s(s^2+1)}$

$\frac{5s}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{0}{s} + \frac{5}{s^2+1}$

$5s = A(s^2+1) + Bs^2 + Cs \quad = \frac{5}{s^2+1} = 5\sin(t)$ ✓

$5s = As^2 + A + Bs^2 + Cs$

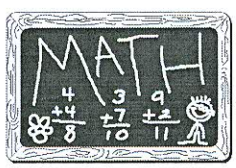
$0 = A + B \Rightarrow 0 = 0 + B$

$\boxed{5 = C}$ $\quad \boxed{B = 0}$

$\boxed{0 = A}$

(20)

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



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A (-1, -1) $f(x,y) = e^{xy}$

B (0, 3)

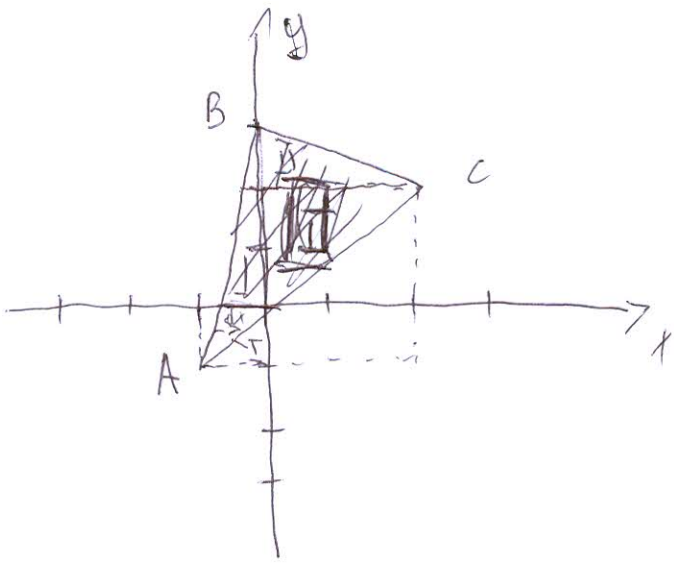
C (2, 2)

$$\iint e^{xy}\,dx\,dy$$

$$\int_{-1}^{T_2}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy + \int_{2}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy$$

$$\int_{-1}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy + \int_{0}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy$$

$$\int_{-1}^{0}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy + \int_{0}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy$$



AB: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y + 1 = \frac{3+1}{0+1}(x+1)$

$y + 1 = 3x + 3$

$y = 3x - 2 \mid \cdot \frac{1}{3}$

$x = \frac{1}{3}y - \frac{2}{3}$

BC: $y - 3 = \frac{2-3}{2-0}(x-0)$

$y - 3 = -\frac{1}{2}x \mid \cdot(-2)$

$-2y + 6 = x$

$x = -2y + 6$

AC: $y + 1 = \frac{2+1}{2+1}(x+1)$

$y + 1 = x + 1$

$y = x$

$\boxed{x = y}$

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