MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: MARIN MARAŠ **BROJ INDEKSA:** 57651

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y)=e^{xy}$. Odrediti $\iint\limits_T f(x,y)\,dx\,dy$. 20
2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$. Izračunati iz definicije $\int\limits_{\partial K}(3-2y)\,ds$. 20
3. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}}(x-y)\,dy$. 20
4. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$x'''(t)+x'(t)=0,\quad x(0)=x''(0)=0,\ x'(0)=5.$$
5. Plohama $x=0$, $y=0$, $z=0$ i $x+y+z=-1$ omeđena je piramida P. Plašt piramide usmjeren prema van označen je sa ∂P. Izračunati $\iint\limits_{\partial P}(z-y)\,dy\,dz$. 20

Ukupno: 20

Tablica integrala

$\int dx = x+C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a}+C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
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$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

Ime i prezime: MARIN MARAŠ   Broj indeksa: 57651

$x'''(t) + x'(t) = 0$ $x(0)=0$, $x''(0)=0$, $x'(0)=5$

$s^3X(s) - s^2\cancel{x(0)}^{0} - s\,x'(0)^{5} - \cancel{x''(0)}^{0} + sX(s) - \cancel{x(0)}^{0} = 0$

$s^3X(s) - 5s + sX(s) = 0$

$s^3X(s) + sX(s) = 5s$

$X(s)(s^3+s) = 5s$

$X(s) = \dfrac{5s}{s(s^2+1)}$

$\dfrac{5s}{s(s^2+1)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1} = \cancel{\dfrac{0}{s}} + \dfrac{5}{s^2+1}$

$5s = A(s^2+1) + Bs^2 + Cs$ $= \dfrac{5}{s^2+1} = 5\sin(t)$ ✓

$5s = As^2 + A + Bs^2 + Cs$

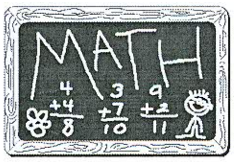
$0 = A + B$ $0 = 0 + B$

$\boxed{5 = C}$ $\boxed{B = 0}$

$\boxed{0 = A}$

20

ZADATKE RIJEŠAVATE <u>JEDNOSTRANO</u> NA OVOM PAPIRU, ALI NA <u>DRUGOJ</u> STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



Ime i prezime: MARIN MARAŠ

Broj indeksa: 57651

$A(-1,-1)$ $\quad f(x,y) = e^{xy}$

$B(0,3)$

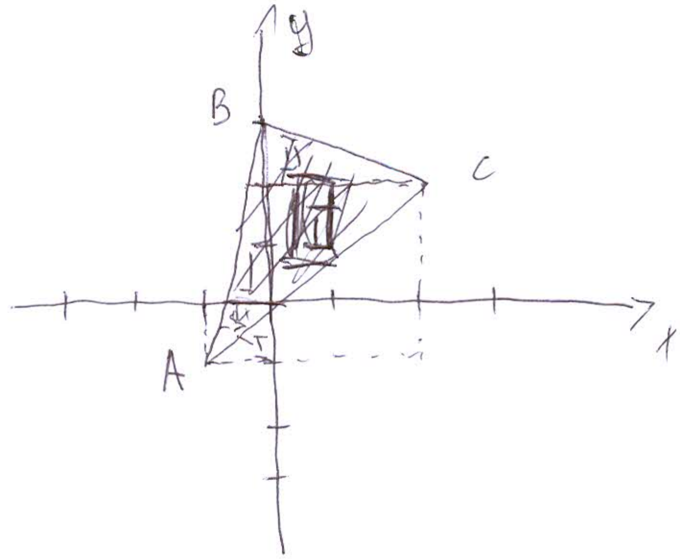
$C(2,2)$

$$\iint e^{xy}\,dx\,dy$$

$$\cancel{\int_{-1}^{2}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy} + \cancel{\int_{2}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy}$$

$$\cancel{\int_{-1}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy} + \cancel{\int_{0}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy}$$

$$\int_{-1}^{0}\int_{\frac{1}{3}y-\frac{2}{3}}^{y} e^{xy}\,dx\,dy + \int_{0}^{3}\int_{\frac{1}{3}y-\frac{2}{3}}^{-2y+6} e^{xy}\,dx\,dy \quad //$$



$$AB: y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y + 1 = \frac{3+1}{0+1}(x+1)$$

$$y + 1 = 3x + 3$$

$$y = 3x - 2 \quad \Big|\cdot \frac{1}{3}$$

$$x = \frac{1}{3}y - \frac{2}{3}$$

$$BC: y - 3 = \frac{2-3}{2-0}(x - 0)$$

$$y - 3 = -\frac{1}{2}x \quad \Big/\cdot(-2)$$

$$-2y + 6 = x$$

$$x = -2y + 6$$

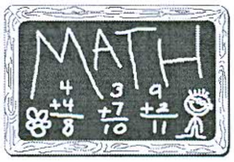
$$AC: y + 1 = \frac{2+1}{2+1}(x+1)$$

$$y + 1 = x + 1$$

$$y = x$$

$$\boxed{x = y}$$

Zadatke riješavate <u>jednostrano</u> na ovom papiru, ali na <u>drugoj</u> strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

IME I PREZIME: ANĐELO UGRINIĆ **BROJ INDEKSA:**

VRIJEME POČETKA: **VRIJEME ZAVRŠETKA:**

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y)=e^{xy}$. Odrediti $\iint\limits_T f(x,y)\,dx\,dy$. 20

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Ukupno:

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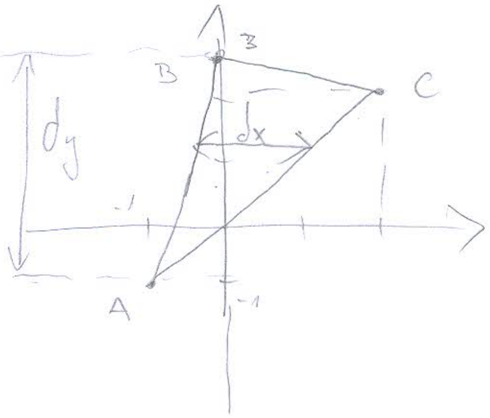
Ime i prezime: ANĐELO UGRINIĆ Broj indeksa: 55581

1\. $A(-1,-1)$, $B(0,3)$, $C(2,2)$

$f(x,y) = e^{xy}$

$\iint\limits_T f(x,y)\,dx\,dy$



$\int_{-1}^{3}\int_{-\frac{1}{4}y+\frac{3}{4}}^{y} dx\,dy = \int_{-1}^{3} dy \int_{-\frac{1}{4}y+\frac{5}{4}}^{y} dx =$

$AB: \quad y+1 = \frac{-1-3}{-1-0}(x+1)$

$y+1 = -4(x+1)$

$y+1 = -4x-4$

$y = -4x-5$

$-4x = y-5 \quad / :-4$

$x = -\frac{1}{4}y+\frac{5}{4}$

$AC: \quad y+1 = \frac{2+1}{2+1}(x+1)$

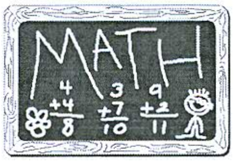
$y+1 = \frac{3}{3}(x+1)$

$y+1 = x+1$

$y = x$

$x = y$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



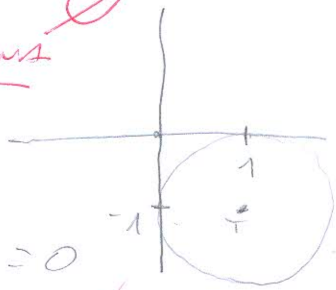
IME I PREZIME: ANĐELO UGRINIĆ BROJ INDEKSA: 55581

(2) $K: \quad r=1 \quad T(1,-1)$

$$\int_{\partial K} (3-2y)\,ds$$

OVDJE JE SKALARNA FUNKCIJA!

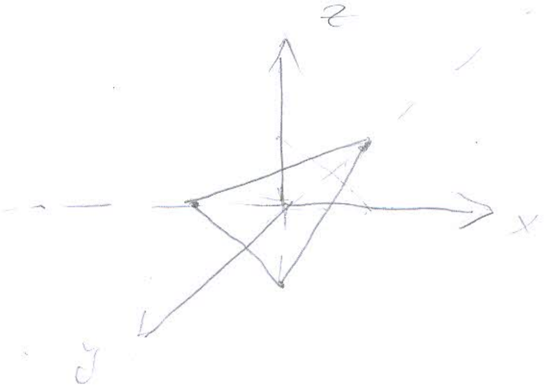
$$W = \begin{bmatrix} 0 \\ 0 \\ 3-2y \end{bmatrix} \Rightarrow \operatorname{div} W = 0+0+0=0$$



$$\int_{\partial K} \operatorname{div} W \,|\, ds = 0$$

(5) $x=0, \; y=0, \; z=0$

$x+y+z=-1$



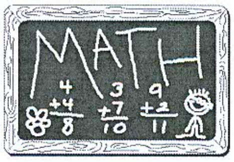
$$\iint_{\partial P} (z-y)\,dy\,dz$$

$$W = \begin{bmatrix} z-y \\ 0 \\ 0 \end{bmatrix} \Rightarrow \operatorname{div} W = 0+0+0=0$$

$$\iint \operatorname{div} W \,|\, ds = 0$$

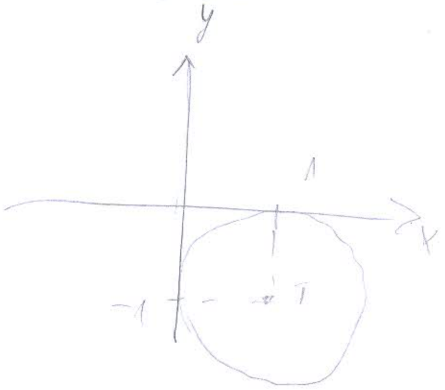
DVOJBENO ?

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



Ime i prezime: ANĐELO UGRINIĆ   Broj indeksa: 55581

(3.) $K : r = 1 \quad T(1, -1)$



$$\oint_{\partial K} (x-y)\,dy$$

$$\int_0^{2\pi}\int_0^{1} r\cos\varphi + 1 - r\sin\varphi - 1$$

$x = r\cos\varphi + 1$

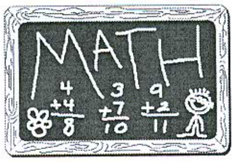
$y = r\sin\varphi - 1$

$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

0

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



Ime i prezime: ANĐELO UGRINIĆ Broj indeksa: 55581

4. $x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 0, \quad x'(0) = 5$

$s^3X(s) - s^2X(0) - sx'(0) - x''(0) + sX(s) - X(0) = 0$

$s^3X(s) - \cancel{s^2 \cdot 0} - s \cdot 5 - \cancel{0} + sX(s) - \cancel{0} = 0$

$s^3X(s) - 5s + sX(s) = 0$

$X(s)(s^3 + s) = 5s$

$X(s) = \frac{5s}{s^3 + s}$

$(s^3 + s) = s(s^2 + 1)$

$(s^2 + 1) = (s-1)(s+1)$

$X(s) = \frac{5s}{s(s-1)(s+1)}$

$\frac{5s}{s(s-1)(s+1)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+1}$

$5s = A(s^2 + 1) + B(s^2 + s) + C(s^2 - s)$

$5s = \underline{As^2} + A \underline{+ Bs^2} \underline{+ Bs +} \underline{Cs^2} - Cs$

$0 = A + B + C$

$5 = B - C$

$0 = A \rightarrow A = 0$

$0 = 0 + B + C$

$0 = B + C$

$5 = B - C$

$5 = 2B$

$2B = 5$

$B = \frac{5}{2}$

$0 = \frac{5}{2} + C$

$\frac{5}{2} = C \Rightarrow C = -\frac{5}{2}$

$X(s) = \frac{5}{2} \cdot \frac{1}{s-1} - \frac{5}{2} \cdot \frac{1}{s+1}$

$x(t) = \frac{5}{2} e^{t} - \frac{5}{2} e^{-t}$

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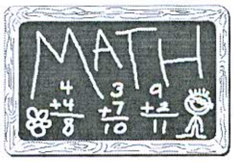

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊ **IME I PREZIME:** ANTE ŠUŠNJARA **BROJ INDEKSA:** 57679 **VRIJEME POČETKA:** 14:15 h **VRIJEME ZAVRŠETKA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y)=e^{xy}$. Odrediti $\iint\limits_T f(x,y)\,dx dy$. 20

2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$. Izračunati iz definicije $\int\limits_{\partial K}(3-2y)\,ds$. 20

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$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

IME I PREZIME: ANTE ŠUŠNJARA BROJ INDEKSA:

2

$r=1$

$T(1,-1)$

$r\in[0,1]$

$\varphi\in[0,2\pi]$

$x = r\cos\varphi + 1$

$y = r\sin\varphi - 1$

$$\int_0^{2\pi}\int_0^1 (3-2(r\sin\varphi-1))r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 (3-(2r\sin\varphi-2))r\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^1 (3r-2r^2\sin\varphi-2r)\,dr\,d\varphi$$

$$\int_0^{2\pi}\left(3\frac{r^2}{2}-2\frac{r^3}{3}\sin\varphi-2\frac{r^2}{2}\right)\Big|_0^1 d\varphi$$

$$\int_0^{2\pi}\left(\frac{3}{2}-\frac{2}{3}\sin\varphi-1\right)d\varphi$$

$$\frac{3}{2}\int_0^{2\pi}d\varphi-\frac{2}{3}\int_0^{2\pi}\sin\varphi\,d\varphi-\int_0^{2\pi}d\varphi$$

$$\frac{3}{2}(2\pi)-\frac{2}{3}\left(-\cos\varphi\Big|_0^{2\pi}\right)-2\pi$$

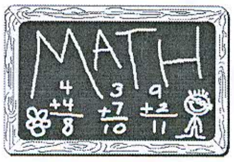
$$\cancel{(-\cos 2\pi-(-\cos 0))}^{0}$$

-1 -1

$= 3\pi - 2\pi$

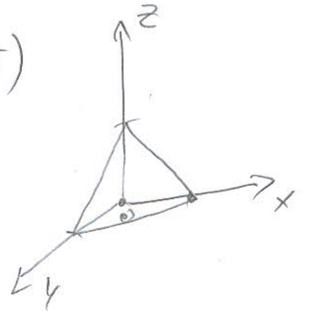
$= \pi$

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.



Ime i prezime: ANTE ŠUŠNJARA Broj indeksa:

5)



$\iint (x-y)\,dy\,dz = \iiint -1\,dx\,dy\,dz$

$= \int_0^{-1}\int_0^{-1-x}\int_0^{-1-x-y} -1\,dz\,dy\,dx$

$= \int_0^{-1}\int_0^{-1-x} -(-1-x-y)\,dy\,dx$

$= \int_0^{-1} 1+x\frac{y^2}{2}\Big|_2^{-1-x} = \int_0^{-1} 1+x+18\,dx$

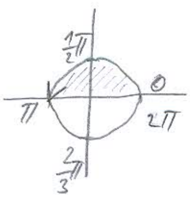
$= 18\int_0^{-1}(1+x)\,dx = -9$

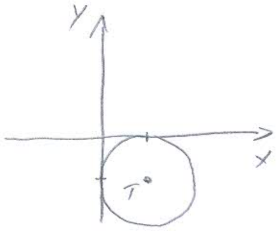
$\emptyset$

3) $r = 1$

$T = (1,-1)$

$\oint_{\partial K}(x-y)\,dy$





$r \in [0, 1]$

$\varphi \in [0, \pi]$

$x = r\cos\varphi + 1$

$y = r\sin\varphi - 1$

$\int_0^{\pi}\int_0^1 -(r\sin\varphi - 1)\,r\,dr\,d\varphi$

$\emptyset$

$\int_0^{\pi}\int_0^1 (-r^2\sin\varphi + r)\,dr\,d\varphi$

$\int_0^{\pi}\left(-\frac{r^3}{3}\sin\varphi\Big|_0^1 + \frac{r^2}{2}\Big|_0^1\right)d\varphi$

$\int_0^{\pi}\left(-\frac{1}{3}\sin\varphi + \frac{1}{2}\right)d\varphi$

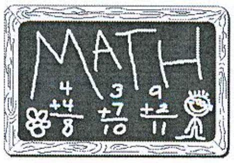
$-\frac{1}{3}\int_0^{\pi}\sin\varphi\,d\varphi + \frac{1}{2}\int_0^{\pi}d\varphi$

$-\frac{1}{3}\left(-\cos\varphi\Big|_0^{\pi}\right) + \frac{1}{2}\left(\varphi\Big|_0^{\pi}\right)$

$-\frac{1}{3}\left(-\cos\pi - (-\cos 0)\right) + \frac{1}{2}\pi$

$-\frac{1}{3}\left(-1 - (-1)\right) + \frac{1}{2}\pi = \frac{1}{2}\pi$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



IME I PREZIME: ANTE ŠUŠNJARA BROJ INDEKSA: 57679

4)

$x'''(t) + x'(t) = 0 \qquad x(0) = x''(0)$

$x'(0) = 5$

$x'''(t) \Rightarrow s^3F(s) - s^2 \cancel{x(0)}^{0} - s\,\cancel{x'(0)}^{5} - \cancel{x''(0)}^{0}$

$x'(t) \Rightarrow s\,F(s) - \cancel{x(0)}^{0}$

$0 = \frac{0}{s} = 0$

$s^3F(s) - 5 + sF(s) = 0$

$s^3F(s) + sF(s) = 5 \qquad s(s^2+1)$

$F(s)(s^3+s) = 5 \,/ :(s^3+s)$

$F(s) = \frac{5}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{(s^2+1)} \,/ s(s^2+1)$

$5 = A(s^2+1) + Bs^2 + Cs$

$5 = As^2 + A + Bs^2 + Cs$

$s^2 \Rightarrow 0 = A + B$

$s^1 \Rightarrow 0 = C$

$s^0 \Rightarrow 5 = A$

$\boxed{A = 5}$

$\boxed{C = 0}$

$A + B = 0$

$B + 5 = 0$

$\boxed{B = -5}$

rješ: $\frac{1}{5} \cdot \frac{1}{s}^{\nearrow 1} + \left(-\frac{1}{5} \cdot \frac{s}{(s^2+1)}^{\nearrow \cos t} + \cancel{\frac{0}{s^2+1}}^{0}\right)$

$= \frac{1}{5} - \frac{1}{5}\cos t$

X

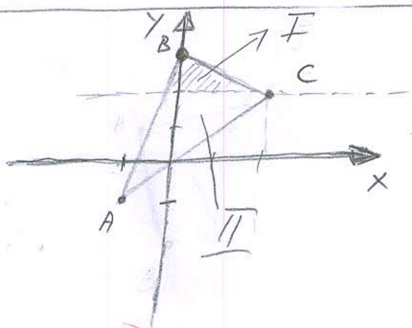
1) $A(-1,-1)$

$B(0, 3)$

$C(2, 2)$

$f(x,y) = e^{xy}$

$\iint_T f(x,y)\,dx\,dy =$



$\int_2^3 \int_{\frac{y-2}{3}}^{-2y} e^{xy}\,dx\,dy + \int_{-1}^{2} \int_{y}^{\frac{y-2}{3}} e^{xy}\,dx\,dy$

0 X

$AB \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y + 1 = \frac{3+1}{0+1}(x+1)$

$y = 3x + 2$ X $y = 4x+3$

$3x = y - 2 \,/ :3$

AB $\boxed{x = \frac{y-2}{3}}$

$BC \Rightarrow y - 0 = \frac{2-3}{2-0}(x - 0)$

$= y = -\frac{1}{2}x \,/ \cdot(-2)$ X

BC $\boxed{x = -2y}$

$AC \Rightarrow y + 1 = \frac{2+1}{2+1}(x+1)$

$y + 1 = x + 1$

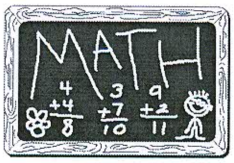
AC $\boxed{x = y}$

$-\frac{1}{2}$

$-\frac{1}{2}$

$-\frac{2}{-2} = 1$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod⇊

POPUNJAVA NASTAVNIK Broj ↓ bodova

IME I PREZIME: MARKO VULELJA **BROJ INDEKSA:** 57660

VRIJEME POČETKA: 14:00 **VRIJEME ZAVRŠETKA:** 15:40

1. Zadan trokut T sa vrhovima: $A(-1,-1)$, $B(0,3)$ i $C(2,2)$ i funkcija $f(x,y)=\mathrm{e}^{xy}$. Odrediti $\iint\limits_T f(x,y)\,\mathrm{d}x\mathrm{d}y$. 20

2. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$. Izračunati iz definicije $\int\limits_{\partial K}(3-2y)\,\mathrm{d}s$. ~~20~~ 0

3. Neka je K krug radijusa $r=1$ sa centrom u točki $T(1,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}}(x-y)\,\mathrm{d}y$. 20

4. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~
$$x'''(t)+x'(t)=0,\quad x(0)=x''(0)=0,\ x'(0)=5.$$

5. Plohama $x=0$, $y=0$, $z=0$ i $x+y+z=-1$ omeđena je piramida P. Plašt piramide usmjeren prema van označen je sa ∂P. Izračunati $\iint\limits_{\partial P}(z-y)\,\mathrm{d}y\mathrm{d}z$. ~~20~~

Ukupno: ~~40~~ 20

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

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④ $x'''(t) + x'(t) = 0$ $\quad x(0) = x''(0)$

$x'(0) = 5$

$f'''(t) \Rightarrow S^3 F(s) - S^2 f(0) - S f'(0) - f(0)$

$f'(t) = SF(s - f(0)$

$S^3 F(s) + SF(s) = 5$

$F(s) = \frac{5}{S(S^2+1)} \Big/ \cdot S(S^2+1)$

$5 = As^2 + A + Bs^2 + Cs$

$0 = A + B \qquad A = 5 \qquad = \frac{1}{5} \cdot \frac{1}{5} - \frac{1}{5} \frac{5}{S^2+1}$

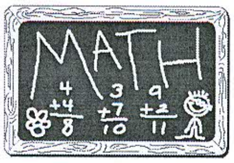
$0 = C \qquad C = 0$

$5 = A \qquad B = -5 \qquad = \frac{1}{5} - \frac{1}{5} \cos t$

$\times$

$x'(0) \neq 5$

Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



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$X(0)=$

$X'(0)$

$S^3X(0)-\cancel{S^2X(0)}+ \dots X(S)-X(0)=0$

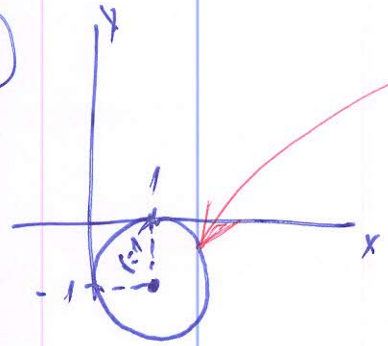
$S^3X(S)-5S+SX(S)=0$

$X(S)=\frac{5}{S^2+1} \dots \frac{1}{S^2+1}$

?

$x(t)=5\sin t$

(3)



$\int (x-y)dy = \iint_K 1dxdy = \pi$

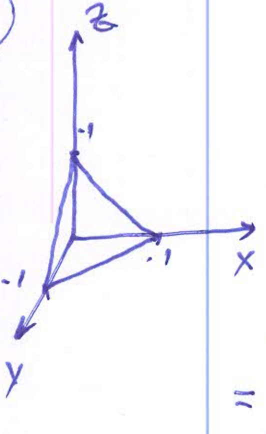
? ✓ 20

$r(t)=(\cos t, \sin t-1)$

$\int_0^{2\pi}$

?

(5.)



$x=0, \; y=0, \; z=0 \quad x+y+z=-1 \quad \partial P$

$\iint_{\partial P} |x-y| dxdz$

$\iint (x-y)dydz = \iint_P -1dxdydz = \int_0^{-1}\int_0^{-1-x}\int_0^{-1-x-y} -1dzdxdy$

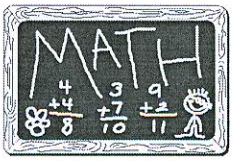
0

$= \int_0^{-1} 1+x\frac{y^2}{2}\Big|_0^{-1-y} = \int_0^{-1} 1+x\frac{(-1-x)^2}{2}dx = \int_0^{-1}\int_0^{-1-y} -(-1-x-y)dydx$

$= \int_0^{-1} 1+x+18\,dx = 18\int_0^{-1}(1+x)dx = -9$

$= \int_0^{-1}\int_0^{-1-y} 1+x+y\,dydx$

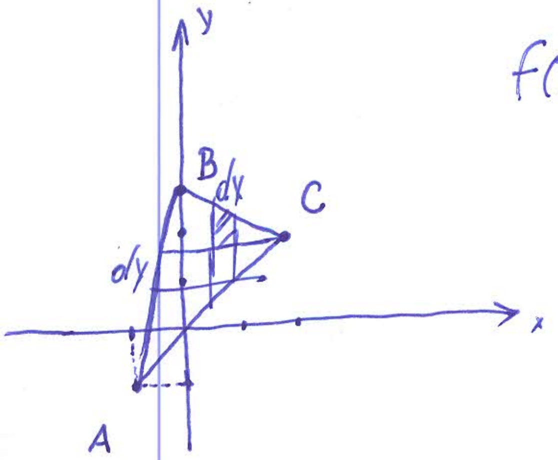
Zadatke riješavate jednostrano na ovom papiru, ali na drugoj strani. Na ovoj strani možete pisati, ali sve što ovdje napišete neće vam biti pregledano niti ocijenjeno.



Ime i prezime: MARKO VULELIJA

Broj indeksa: 57660

1.

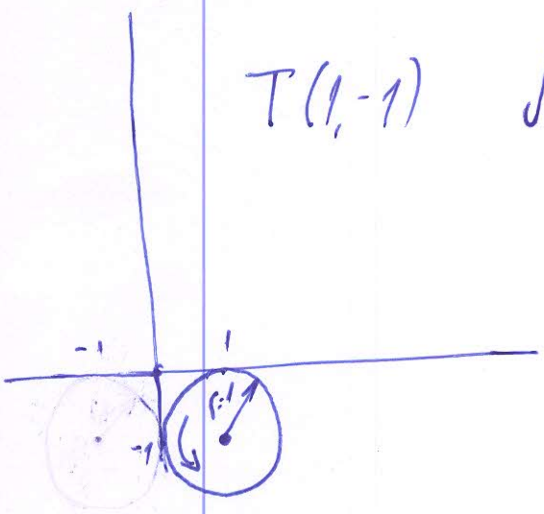


$f(x,y)=e^{xy}$

$$\iint_T f(x,y)\,dx\,dy$$

?

2.



$T(1,-1)$ $\quad \int (3-2y)\,ds$

$r(t)=(\cos t+1,\ \sin t-1)$

$r(t)=(\cos t,\ \sin t)$

$r'(t)=(-\sin t,\ \cos t)$

$=1$

$$\int_{\partial K}(3-2y)\,ds=\int_0^{2\pi}(3-2\sin t)\cdot dt=6\pi$$

$$=\int_0^{2\pi} 3-2(\sin t-1)\cdot dt$$

~~20~~

0

ZADATKE RIJEŠAVATE JEDNOSTRANO NA OVOM PAPIRU, ALI NA DRUGOJ STRANI. NA OVOJ STRANI MOŽETE PISATI, ALI SVE ŠTO OVDJE NAPIŠETE NEĆE VAM BITI PREGLEDANO NITI OCIJENJENO.