

**MATEMATIKA I - KOLOKVIJ #1:**

**PRAVILA** Studentima koji posjeduju mobilni telefon treba biti ugašen. Studentima na ispitu nisu dopuštene nikakve formule. Nikakvo posuđivanje pribora nije dopušteno. U vrijeme trajanja ispita studenti ne mogu izlaziti van bez predaje ispita. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

**TRAJANJE: 45 MINUTA. PIŠITE DVOSTRANO!** Obavezno popuniti sva polja ispod. U pitanjima s višestrukim ponuđenim odgovorima može biti više točnih.

IME I PREZIME: TIN LOBOREC

BROJ INDEKSA: 17-2-0108-2012

VRIJEME POČETKA: 09:10

VRIJEME ZAVRŠETKA: 10:16

POPUNJAVA

NASTAVNIK

Broj ↓  
bodova

Ukupno:

8

1. Po definiciji funkcija se sastoji od:

- (a) grafa
- (b) domene
- (c) kodomene
- (d) inverza
- (e) slike
- (f) pravila
- (g) ništa od navedenog

2. Rang matrice ne može biti manji od:

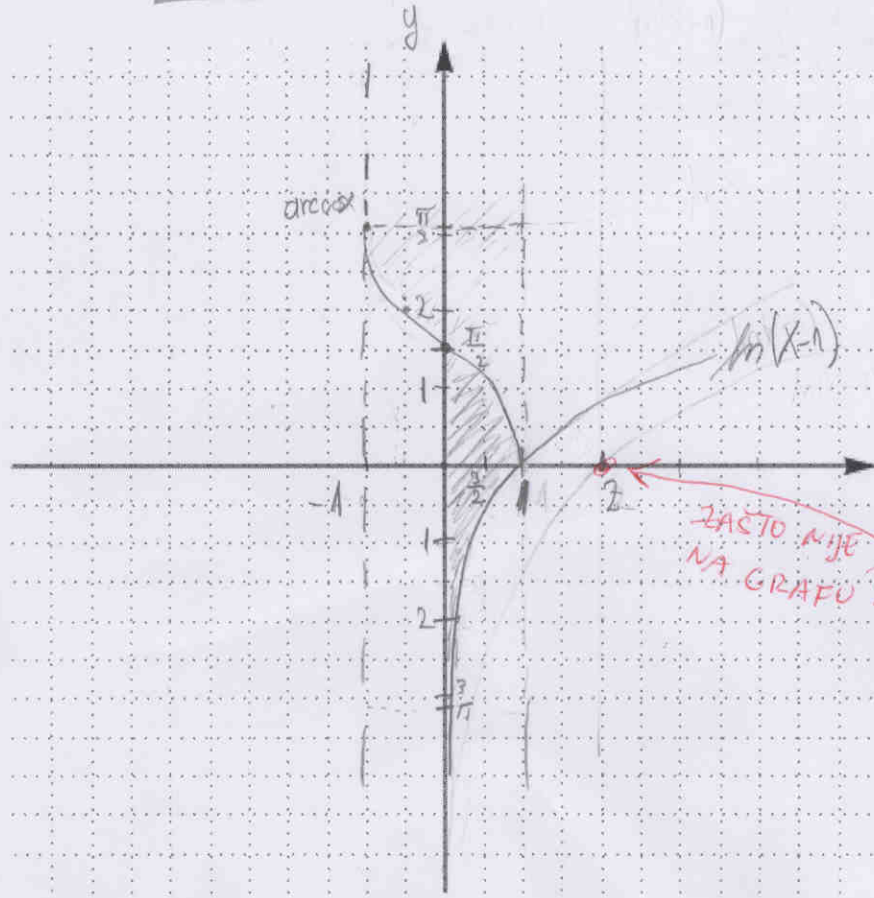
- (a) broja stupaca
- (b) broja redaka
- (c) ništa od navedenog

3. Zaokružiti sve neparne funkcije: kvadratna, kubna, drugi korijen, treći korijen, eksponenci-  
jalna, logaritamska, sinus, arkus sinus, kosinus, arkus kosinus, tangens, arkus tangens

4. Grafički odrediti rješenja nejednadžbe:  $\ln(x-1) < \arccos x$

za  $\ln \Rightarrow x > 0$

Projera: mpr.  $x = \frac{3}{2}$   
 $\ln(\frac{3}{2}-1) < \arccos \frac{3}{2}$   
 $\ln(\frac{1}{2}) < \arccos \frac{3}{2}$   
 $-0,69 < 1,0472$



$\arccos x \Rightarrow$   
 $[-1, 1]$

| x  | y               |
|----|-----------------|
| -1 | $\frac{\pi}{2}$ |
| 0  | $\frac{\pi}{2}$ |
| 1  | 0               |

$\ln(x-1) = y$

| x | y      |
|---|--------|
| 1 | 0      |
| 2 | 0,69   |
| 3 | 1,0986 |

ZASTO NIJE NA GRAFU?

5. Riješiti u skupu realnih brojeva:  $e^{\sqrt{x}} = 0.7$

$$e^x \neq 0$$

$$e^{\sqrt{x}} = 0.7 / \ln$$

$$\sqrt{x} = \ln 0.7$$

$$\sqrt{x} = -0.356 / 2$$

$$x = 0.1272 \quad \times$$

$$\sqrt{0.1272} = 0.356 \neq -0.356$$

6. Riješiti jednačbu u kompleksnim brojevima:  $z^3 + i = 2 + i$

$$z^3 + i = 2 + i \quad \rightarrow \text{logaritiramo kompleksni}$$

$$z^3 + i = 2 - i$$

$$z^3 = 2 + 2i \quad \left( \sqrt[3]{\phantom{x}} \right)$$

$$z = \sqrt[3]{2 + 2i}$$

DAJE...?

$$z = \underbrace{x}_{\text{Re}} + \underbrace{yi}_{\text{Im}}$$

7. Gaussovom metodom riješi sustav linearnih jednačini:

8

$$\begin{aligned} \frac{5}{2}x + y - 7z - 7u &= 5 \\ 5x + 4y + 5u &= 5 \\ -2y + 5z + 7u &= 0 \\ -7x - 3y - 7u &= -7 \end{aligned}$$

$$\left[ \begin{array}{cccc|c} \frac{5}{2} & 1 & -7 & -7 & 5 \\ 5 & 4 & 0 & 5 & 5 \\ 0 & -2 & 5 & 7 & 0 \\ -7 & -3 & 0 & -7 & -7 \end{array} \right] \left| \cdot \left( \frac{2}{5} \right) \right.$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 5 & 4 & 0 & 5 & 5 \\ 0 & -2 & 5 & 7 & 0 \\ -7 & -3 & 0 & -7 & -7 \end{array} \right] \begin{array}{l} -5 \cdot \text{I.r.} \\ +7 \cdot \text{I.r.} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 2 & 14 & 24 & -8 \\ 0 & -2 & 5 & 7 & 0 \\ 0 & \frac{1}{5} & -\frac{98}{5} & \frac{133}{5} & 7 \end{array} \right] \begin{array}{l} + \text{II.r.} \\ \cdot \left( \frac{5}{1} \right) \end{array}$$

$4 - 2 = 2 \checkmark$   
 $0 + 8 \left( \frac{14}{5} \right) \quad 5 + 5 \left( \frac{24}{5} \right)$

$-3 + \frac{14}{5} = \frac{-15 + 14}{5}$

$0 - \frac{98}{5} \quad -7 - \frac{98}{5} = \frac{-35 - 98}{5}$

$$\left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 2 & 14 & 24 & -8 \\ 0 & 0 & 19 & 31 & -8 \\ 0 & 1 & 98 & 133 & -35 \end{array} \right] \begin{array}{l} -2 \cdot \text{II.r.} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 2 & 14 & 24 & -8 \\ 0 & 0 & 19 & 31 & -8 \\ 0 & 0 & 70 & 85 & -27 \end{array} \right] \begin{array}{l} \cdot \frac{1}{19} \\ \cdot \frac{1}{19} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 1 & 7 & 12 & -4 \\ 0 & 0 & 19 & 31 & -8 \\ 0 & 0 & 70 & 85 & -27 \end{array} \right] \left( \frac{1}{19} \right)$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 1 & 7 & 12 & -4 \\ 0 & 0 & 1 & \frac{31}{19} & -\frac{8}{19} \\ 0 & 0 & 0 & \frac{145}{19} & -\frac{224}{19} \end{array} \right] \begin{array}{l} + \text{III.r.} \\ + \text{III.r.} \end{array}$$

$$\left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 1 & 7 & 12 & -4 \\ 0 & 0 & 1 & \frac{31}{19} & -\frac{8}{19} \\ 0 & 0 & 0 & \frac{145}{19} & -\frac{224}{19} \end{array} \right] \begin{array}{l} \cdot \frac{19}{145} \\ \cdot \frac{19}{145} \end{array}$$

$$\sim \left[ \begin{array}{cccc|c} 1 & \frac{2}{5} & -\frac{14}{5} & -\frac{14}{5} & 2 \\ 0 & 1 & 7 & 12 & -4 \\ 0 & 0 & 1 & \frac{31}{19} & -\frac{8}{19} \\ 0 & 0 & 0 & 1 & -\frac{16}{19} \end{array} \right] \begin{array}{l} \cdot \frac{19}{145} \\ \cdot \frac{19}{145} \end{array}$$

$-27 + 3 \left( \frac{145}{19} \right) = \frac{-513 + 435}{19} = \frac{-78}{19}$

# 7. ZADATAK!

Ako vam nedostaje mjesta za neki zadatak slobodno nastavite pisati ovdje (samo istaknite broj zadatka).

$$\begin{bmatrix} 5 & 1 & -7 & -7 & 5 \\ 2 & 5 & 4 & 0 & 5 \\ 0 & 2 & 5 & 7 & 0 \\ -7 & -3 & 0 & -7 & -7 \end{bmatrix} \xrightarrow{I: \cdot (-7)} \begin{bmatrix} 5 & 1 & -7 & -7 & 5 \\ 5 & 4 & 0 & 5 & 5 \\ 0 & -2 & 5 & 7 & 0 \\ 1 & \frac{3}{7} & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-5I} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 1 & 1 \\ 5 & 4 & 0 & 5 & 5 \\ 0 & -2 & 5 & 7 & 0 \\ \frac{5}{2} & 1 & -7 & -7 & 5 \end{bmatrix} \xrightarrow{-\frac{5}{2}I} \begin{bmatrix} 1 & \frac{3}{7} & 0 & 1 & 1 \\ 0 & \frac{13}{7} & 0 & 0 & 0 \\ 0 & -2 & 5 & 7 & 0 \\ 0 & \frac{1}{14} & -7 & \frac{11}{2} & \frac{5}{2} \end{bmatrix} \xrightarrow{I: \cdot \frac{7}{13}}$$

$$\begin{bmatrix} 1 & \frac{3}{7} & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 5 & 7 & 0 \\ 0 & \frac{1}{14} & -7 & \frac{11}{2} & \frac{5}{2} \end{bmatrix} \xrightarrow{-\frac{3}{7}II, \text{REDKA}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & -7 & \frac{13}{2} & \frac{15}{2} \end{bmatrix} \xrightarrow{+2III, \text{RED}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & -7 & \frac{13}{2} & \frac{15}{2} \end{bmatrix} \xrightarrow{+I, \text{RED}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 5 & 7 & 0 \\ 0 & 0 & -7 & \frac{13}{2} & \frac{15}{2} \end{bmatrix} \xrightarrow{I: \cdot \frac{1}{5}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & -7 & \frac{13}{2} & \frac{15}{2} \end{bmatrix} \xrightarrow{+7III, \text{RED}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & -7 & \frac{13}{2} & \frac{15}{2} \end{bmatrix} \xrightarrow{I: \cdot \frac{1}{10}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & 0 & \frac{3}{10} & \frac{5}{2} \end{bmatrix} \xrightarrow{I: \cdot \frac{10}{3}} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{25}{3} \end{bmatrix} \xrightarrow{-\frac{13}{2} + \frac{49}{5} = \frac{-35+98}{10}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{25}{3} \end{bmatrix} \xrightarrow{-IV} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{7}{5} & 0 \\ 0 & 0 & 0 & 1 & \frac{25}{3} \end{bmatrix} \xrightarrow{-\frac{7}{5}IV} \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-22}{3} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{-35}{3} \\ 0 & 0 & 0 & 1 & \frac{25}{3} \end{bmatrix} \begin{matrix} X = \frac{-22}{3} \\ Y = 0 \\ Z = \frac{-35}{3} \\ M = \frac{25}{3} \end{matrix} \begin{bmatrix} X \\ Y \\ Z \\ M \end{bmatrix} = \begin{bmatrix} \frac{-22}{3} \\ 0 \\ \frac{-35}{3} \\ \frac{25}{3} \end{bmatrix}$$

Provjera rješenja.

$$\frac{5}{2} \cdot \left(\frac{-22}{3}\right) + 0 + \frac{245}{3} - \frac{1175}{3} = \frac{-110}{6} + \frac{245}{3} - \frac{1175}{3} = \frac{-110 + 490 - 350}{6} = \frac{-70}{6} = -\frac{35}{3} \neq -7$$

$$\frac{-110}{3} + 0 + 0 + \frac{1175}{3} = \frac{1065}{3} = 355 \neq 5$$

$$0 - 0 - \frac{1175}{3} + \frac{1175}{3} = 0 = 0 \checkmark$$

$$\frac{154}{3} - 0 + 0 - \frac{1175}{3} = \frac{154 - 1175}{3} = \frac{-1021}{3} \neq -7$$

