MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. IME I PREZIME: MARINO PRENDŽA BROJ INDEKSA: 57659	Grupa xxoox POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) — 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2, 1)$. Izraèunati $\iint_{K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. — ~~20~~ 15

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — ~~20~~

$$f'''(t) + f'(t) = 1, \quad x(0) = 1,\ x'(0) = 1,\ x''(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0, -1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x + 3)\,\mathrm{d}y$. — ~~20~~

5. Provjeri da li je $\mathbf{w}(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x, y, z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. — ~~20~~

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

3. $f'''(t) + f'(t) = 1 \qquad x(0) = 1,\ x'(0) = 1,\ x''(0) = 1$

$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + (sF(s) - f(0)) = 1 \quad \times \quad \frac{1}{s}$

$s^3F(s) - s^2 - s - 1 + sF(s) - 1 = 1$

$s^3F(s) - s^2 - s - 2 + sF(s) = 1$

$F(s)(s^3 + s) = 1 + s^2 + s + 2$

$F(s)(s^3 + s) = s^2 + s + 3 \;/ : (s^3 + s)$

$F(s) = \dfrac{s^2 + s + 3}{s^3 + s}$

$F(s) = \dfrac{s^2 + s + 3}{s(s^2 + 1)}$

$$\frac{s^2+s+3}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$$

$$s^2+s+3 = As^2 + A + Bs^2 + Cs$$

$$s^2+s+3 = s^2(A+B) + Cs + A$$

$$A+B=1 \rightarrow 3+B=1$$

$$\boxed{C=1} \qquad B = 1-3$$

$$\boxed{A=3} \qquad \boxed{B=-2}$$

$$F(s) = 3\cdot\frac{1}{s} - 2\cdot\frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$$F(t) = \mathcal{L}^{-1}\left\{3\cdot\frac{1}{s} - 2\cdot\frac{s}{s^2+1} + \frac{1}{s^2+1}\right\}$$

$$f(t) = 3 - 2\cos(t) + \sin(t)$$

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2. KROG $\quad r=1 \quad T(2,1)$

$$\iint_K (2x+3)\,dx\,dy$$

$$x = r\cos\varphi + 2 \quad ✓$$
$$y = r\sin\varphi + 1 \quad ✓$$

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$$r \in [0,1]$$
$$\varphi \in [0, 2\pi]$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2(r\cos\varphi+2)+3)\,r\,dr = \int_0^{2\pi} d\varphi \int_0^1 (2r\cos\varphi + 5)\,r\,dr =$$

$$= \int_0^{2\pi} d\varphi \int_0^1 2r^2\cos\varphi + 5r\,dr = \int_0^{2\pi} d\varphi \left(2\cdot\frac{r^3}{3}\cos\varphi + 5\cdot\frac{r^2}{2}\right)\Bigg|_0^1 =$$

$$= \int_0^{2\pi} \left(2\cdot\frac{1}{3}\cos\varphi + 5\cdot\frac{1}{2}\right) d\varphi = \int_0^{2\pi} \frac{2}{3}\cos\varphi + \frac{5}{2}\,d\varphi = \frac{2}{3}\sin\varphi + \frac{5}{2}\varphi\Bigg|_0^{2\pi} =$$

$$= \cancel{\frac{2}{3}\sin 2\pi}^{0} + \frac{5}{2}\cdot 2\pi - \left(\cancel{\frac{2}{3}\sin 0}^{0} + \cancel{\frac{5}{2}\cdot 0}^{0}\right) = \frac{5}{\cancel{2}}\cdot\cancel{2}\pi = 5\pi$$

✓ ×

5. $\vec{w}(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

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$x = 1+2\cos t \quad y = 1-3\sin t \quad z = 1-3\sin t$

$t \in [0, 2\pi]$

$\int_\Gamma (w | dr) \qquad r'(t) = \begin{bmatrix} -2\sin t \\ -3\cos t \\ -3\cos t \end{bmatrix}$

DA LI JE W POTENCIJALNO POLJE ?

$\varnothing$

$\|r'(t)\| = \sqrt{(-2\sin t)^2 + (-3\cos t)^2 + (-3\cos t)^2}$

$\|r'(t)\| = \sqrt{4\sin^2 t + 9\cos^2 t + 9\cos^2 t}$

$\|r'(t)\| = \sqrt{4\sin^2 t + 18\cos^2 t}$

$$W(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2+z^2}} \\ \frac{y}{\sqrt{x^2+y^2+z^2}} \\ \frac{z}{\sqrt{x^2+y^2+z^2}} \end{pmatrix}$$

$$\frac{x}{\sqrt{x^2+y^2+z^2}} = \frac{1+2\cos t}{\sqrt{(1+2\cos t)^2 + (1-3\sin t)^2 + (1-3\sin t)^2}} =$$

$$= \frac{1+2\cos t}{\sqrt{1+4\cos^2 t + 1 + 9\sin^2 t + 1 + 9\sin^2 t}} = \frac{1+2\cos t}{\sqrt{4\cos^2 t + 3 + 18\sin^2 t}}$$

?

4. $r \in [0,1]$ $T(0,-1)$

$\int_0^1 (2x+3)\,dy$

$\cancel{0}$