MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	Grupa xxoox
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1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\,\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2, 1)$. Izraèunati $\iint\limits_{K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. ~~20~~ 15

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1,\ x'(0) = 1,\ x''(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0, -1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x + 3)\,\mathrm{d}y$. 20

5. Provjeri da li je $\mathbf{w}(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x, y, z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: 35

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

OKRENI

③ $f'''(t) + f'(t) = 1$ $\qquad x(0) = 1$, $x'(0) = 1$, $x''(0) = 1$

$$s^3 F(s) - s^2 \overbrace{f(0)}^{1} - s \overbrace{f'(0)}^{1} - \overbrace{f''(0)}^{1} + s F(s) - \overbrace{f(0)}^{1} = \frac{1}{s}$$

$$F(s)\Big(\overbrace{s^3 + s}^{s(s^2+1)}\Big) = \frac{1}{s} + s^2 + s + 2$$

$$F(s)(s^3+s) = \frac{s^3+s^2+2s+1}{s} \quad / : (s^3+s) = s(s^2+1)$$

$$F(s) = \frac{s^3+s^2+2s+1}{s^2(s^2+1)}$$

$$\frac{s^3+s^2+2s+1}{s^2(s^2+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs+D}{s^2+1}$$

$$s^3+s^2+2s+1 = A(s^2+1) + B(s^3+s) + (Cs+D)\,s^2$$

$$s^3+s^2+2s+1 = As^2 + A + Bs^3 + Bs + Cs^3 + Ds^2$$

$1 = B + C \Rightarrow -C = B - 1$; $C = -B + 1$; $\boxed{C = -1}$

$1 = A + D$ $\qquad$ $1 = A + D$; $1 = 1 + D$; $\boxed{D = 0}$

$2 = B \Rightarrow \boxed{B = 2}$

$1 = A \Rightarrow \boxed{A = 1}$

$$F(s) = \frac{1}{s^2} + 2 \cdot \frac{1}{s} - \frac{s}{s^2+1}$$

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{s^2} + 2 \cdot \frac{1}{s} - \frac{s}{s^2+1}\right]$$

$$f(t) = t + 2 - \cos(t)$$

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(2) $r = 1 \qquad T(2,1)$

$$dx\,dy = r\,dr\,d\varphi$$

$$x = r\cos\varphi + 2$$

$$y = r\sin\varphi + 1$$

$$\varphi \in [0, 2\pi]$$

$$r \in [0, 1]$$

$$\iint_K (2x+3)\,dx\,dy$$

X

$$\iint_K \big(2(r\cos\varphi + \overbrace{2) + 3}^{5}\big)\,r\,dr\,d\varphi$$

$$= 2\int_0^{2\pi}\int_0^1 r^2\cos\varphi\,dr\,d\varphi + \overset{7}{5}\int_0^{2\pi}\int_0^1 r\,dr\,d\varphi$$

$$= 2\int_0^{2\pi}\left(\frac{r^3}{3}\right)\cos\varphi\,d\varphi\,\Big|_0^1 + \overset{7}{5}\int_0^{2\pi}\left(\frac{r^2}{2}\right)d\varphi\,\Big|_0^1$$

$$= \frac{2}{3}\int_0^{2\pi}\cos\varphi\,d\varphi + \frac{\overset{7}{5}}{2}\int_0^{2\pi}d\varphi$$

$$= \frac{2}{3}\sin\varphi\,\Big|_0^{2\pi} + \frac{\overset{7}{5}}{2}\varphi\,\Big|_0^{2\pi}$$

$$= \cancel{\frac{2}{3}\left(\overset{0}{\sin(2\pi)} - \overset{0}{\sin(0)}\right)} + \frac{2\pi\,\overset{7}{5}}{\cancel{2}}$$

$$= \overset{7}{5}\pi \approx 15{,}708$$

1,5