MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\,dx\,dy$. (pomoć: $\operatorname{rot}(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint_K (2x+3)\,dx\,dy$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1, x'(0) = 1, x''(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,dy$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|d\mathbf{r})$. 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(3) $f'''(t) + f'(t) = 1$ $\quad x(0)=1,\ x'(0)=1,\ x''(0)=1$

$s^3X(s) - s^2x(0) - sf'(0) - f''(0) + sX(s) - f(0) = \frac{1}{s}$

$s^3X(s) - s^2 - s - 1 + sX(s) - 1 = \frac{1}{s}$

$s^3X(s) + sX(s) = \frac{1}{s} + s^2 + s + 2$

$X(s)(s^3+s) = \frac{s^2+s+3}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}$

$s^2+s+3 = A(s^2+1) + Bs^2 + Cs$

$s^2+s+3 = As^2 + A + Bs^2 + Cs$

$1 = A + B$ $\quad B = -2$

$1 = C$ $\quad C = 1$

$3 = A$ $\quad A = 3$

$= 3 - \frac{1}{2}\frac{s}{s^2+1}$

$= 3 - \frac{1}{2}\cos t$

VIDI VIŠE

$f'''(t) + f'(t) = 1$

$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0)$

$s^3F(s) - s^2 - s - 1 + (sF(s) - 1) = \frac{1}{s}$

$s^3F(s) + sF(s) = 1 + s^2 + s + 1 + 1$

$s^3F(s) + sF(s) = s^2 + s + 3$

$F(s)(s^3 + s) = s^2 + s + 3 \quad / : (s^3 + s)$

$F(s) = \frac{s^2 + s + 3}{s(s^2 + 1)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1}$

$s^2 + s + 3 = A(s^2 + 1) + Bs^2 + Cs$

$s^2 + s + 3 = As^2 + A + Bs^2 + Cs$

$1 = A + B \qquad 1 = 3 + B$

$1 = C \qquad -2 = B$

$3 = A \qquad \boxed{A = 3} \qquad \boxed{C = 1}$

$X(s) = \frac{3}{s} - \frac{1}{2}\cos t$ ✗