MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\,dx dy$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint_K (2x+3)\,dx dy$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1,\ x'(0) = 1,\ x'(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,dy$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|d\mathbf{r})$. 20

Ukupno: 15

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

KURILIĆ

③ $f'''(t) + f'(t) = 1$, $x(0)=1$, $x'(0)=1$, $x''(0)=1$

$f''' \to s^3 X(s) - s^2 \underbrace{x(0)}_{1} - s^1 \underbrace{x'(0)}_{1} - \underbrace{x''(0)}_{1}$

$\boxed{f''' \to s^3 x(s) - s^2 - s - 1}$

$f' \to s^1 x(s) - \underbrace{x(0)}_{1} \Rightarrow \boxed{s\,x(s) - 1}$

$s^3 x(s) - s^2 - s - 1 + s x(s) - 1 = 1$

$s^3 x(s) + s x(s) = s^2 + s + 1 + 1 + 1$

$s^3 x(s) + s x(s) = s^2 + s + 3$

$x(s)(s^3 + s) = s^2 + s + 3$

$x(s) = \frac{s^2+s+3}{s^3+s}$

$x(s) = \frac{s^2+s+3}{s(s^2+1)}$

$\frac{s^2+s+3}{} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{(s^2+1)} \quad / \cdot s(s+1)(s^2+1)$

$= A(s+1)(s^2+1) + B(s^2+1)s + (Cx+D)(s+1)s$

$= A(s^3+s+s^2+1) + B(s^3+s) + (Cs+D)(s^2+s)$

$= \underline{As^3} + \underline{As} + \underline{As^2} + \underline{A} + \underline{Bs^3} + \underline{Bs} + \underline{Cs^3} + \underline{Cs^2} + \underline{Ds^2} + \underline{Ds}$

(s^3) $0 = A + B + C$

(s^2) $1 = A + C + D \Rightarrow 1 = A + C + 0$

(s^1) $1 = A + B + D$ $\quad 1 = 3 + C + 0$

(s^0) $3 = A \Rightarrow \boxed{A=3}$ $\quad -C = -1 + 3 + 0$

$-C = 2 + 0$

$C = -2 - D$

$C = -2 - 1$

$\boxed{C = -3}$

3. ZADATAK
NASTAVAK

$0 = A + B + C$

$-B = A + C$

$-B = 3 + C$

$-B = 3 - 2 - D$

$-B = 1 - D$

$\boxed{B = D - 1}$

$\Downarrow$

$B = 1 - 1$

$\boxed{B = 0}$

$1 = A + B + D$

$1 = 3 + D - 1 + D$

$1 = 3 + 2D - 1$

$1 = 2 + 2D$

$-2D = 2$

$2D = -2$

$\boxed{D = 1}$

$$f(s) = \frac{3}{s} + \frac{0}{s+1} - \frac{3s+1}{(s^2+1)}$$

$$f(s) = \frac{3}{s} + 0 - \frac{3s}{(s^2+1)} + \frac{1}{(s^2+1^2)}$$

$$\boxed{f(t) = 3 - (1-2t)e^{-2t} + te^{-2t}}$$ ✗

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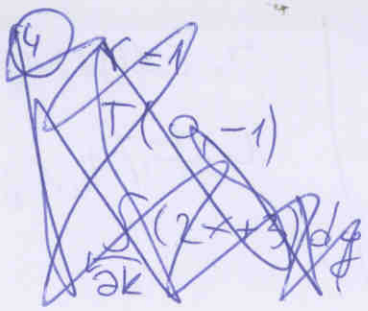

(2) $r=1$

$T(2,1)$

$\iint_K (2x+3)\,dx\,dy$

$x = r\cos\varphi + p \Rightarrow \boxed{x = r\cos\varphi + 2}$ ✓

$y = r\sin\varphi + q \Rightarrow \underline{x = r\sin\varphi + 1}$ ✓

$$\int_0^{2\pi} d\varphi \int_0^1 (2x+3)\, r\, dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 (2x+3)\, r\, dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 [2(r\cos\varphi+2)+3]\, r\, dr$$ ✓ 15

$$\int_0^{2\pi} d\varphi \int_0^1 [2r\cos\varphi + 7]\, r\, dr$$

$$\int_0^{2\pi} d\varphi \int_0^1 2r^2\cos\varphi + 7\int_0^1 r\, dr$$

$$\int_0^{2\pi} d\varphi \cos\varphi \, 2\int_0^1 r^2\, dr + 7\int_0^1 r\, dr$$

$$\int_0^{2\pi} d\varphi \cos\varphi \left(2\frac{r^3}{3}\Big|_0^1 + 7\frac{r^2}{2}\Big|_0^1\right)$$

$$\int_0^{2\pi} d\varphi \cos\varphi \left(2\cdot\frac{1^3}{3} - 0\right) + \left(7\frac{1^2}{2} - 0\right)$$

$$\int_0^{2\pi} d\varphi \cos\varphi \, \frac{2}{3} + \frac{7}{2}$$ ✓ $\Rightarrow \int_0^{2\pi} d\varphi \cos\varphi \frac{4+21}{6}$ ✗

$$\frac{25}{6}\int_0^{2\pi} d\varphi \cos\varphi \Rightarrow \frac{25}{6}\sin\varphi\Big|_0^{2\pi} \Rightarrow \frac{25}{6}(\sin 2\pi - \sin 0)$$

$$= \frac{25}{6}(0-0) = 0$$

(5) $w(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = F'(t) \begin{bmatrix} \frac{x}{\sqrt{x^2+y^2+z^2}} \\ \frac{y}{\sqrt{x^2+y^2+z^2}} \\ \frac{z}{\sqrt{x^2+y^2+z^2}} \end{bmatrix}$

?

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(1) $r=1 \quad (z \geq 0) \qquad \iint_{\partial K} 3\,dx\,dy$

$$\int_0^{\frac{\pi}{2}} d\varphi \int_0^1 r\cos\varphi +$$

$$x = r\cos\varphi + p$$
$$y = r\sin\varphi + q$$

$x - p$