MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\mathrm{d}x\mathrm{d}y$. (pomoć: $\operatorname{rot}(3x\mathbf{j}) = 3\mathbf{k}$) ~~20~~

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y$. ~~20~~

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1,\ x'(0) = 1,\ x'(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int\limits_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. ~~20~~

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: ~~0~~

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1.) $\iint\limits_{\partial K} 3\,dx\,dy = \int_3^1 3x\,dx \int_3^1 3y\,dy = \int_2^1 3y\,dy \int 3x\,dx + \int_3^1 3y\,dy \int_3^1 3x\,dx$ ~~0~~

$= \int_2^1 3y\,dy\,(\ln x)\Big|_2^y + \int_3^1 3x\,dx\,(\ln x)\Big|_2^1 = \int_3^1 y(\ln y - \ln x)\,dy + \int_3^1 x(\ln x - \ln)\,dx$

$= \int_3^1 \ln y^2 - \frac{1\,dx}{y}\,dy + \int_2^1 \ln x^2 - \frac{1\ln}{x}\,dx = 6\ln y - \frac{dx}{y}\,dy + 6\ln x - \frac{\ln x}{x}\,dx$

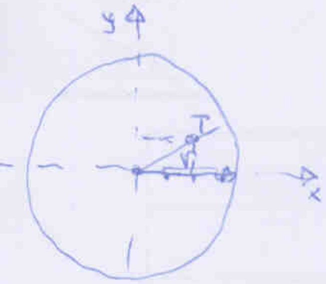
$= 12$

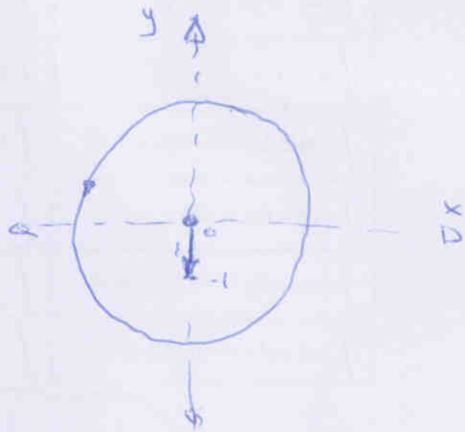
2) $\iint_K (2x+3)\,dx\,dy = \int_1^2 2x\,dx \int 3y\,dy = \int_1^2 3y\,dy \int_1^y 2x\,dx + \int_2^1 3y\,dy \int_2^1 2x\,dx$

$= \int_1^2 3y\,dy\,\left|\ln x\right|_2^y + \int_2^2 y\,dy\,\left|\ln x\right|_2^1 = \int_1^1 y(\ln y - \ln 1)\,dy + \int_2^2 y(\ln 2 - \ln y)\,dy =$

$= \left|\frac{1}{2}3y\ln y - \frac{1}{2}y^2 - 3y^2\ln 1\right|_2^1 + \left|3y^2\ln 1 - 1y^2\ln y + \frac{1}{2}y^2 + 2y^2\ln 2\right| =$

$=$





4) $r = 1$, $T(0,-1)$

$\oint_K (2x+3)\,dy = \int_{-1}^0 2x\,dy + \int_{-1}^0 3y\,dy = \int_{-1}^0 2x\,dy + \int_{-1}^0 3y\,dy^2$

$\int_{-1}^0 2x + \int_{-1}^0 3y\,dy^2 = \int_{-1}$