MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\,\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint_K (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1, x'(0) = 1, x''(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: 20

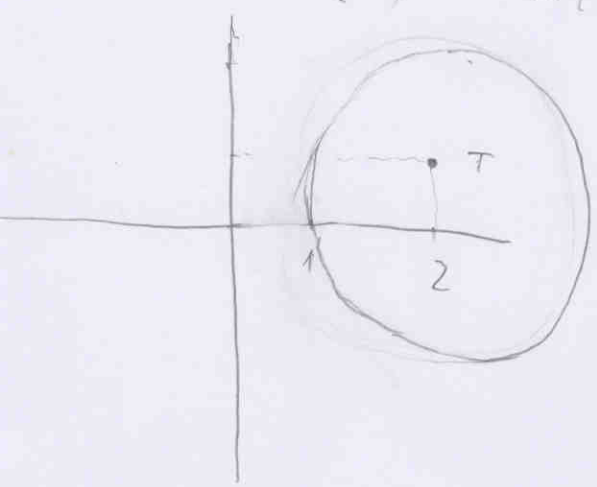
Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

2. $r=1$ $T(2,1)$ $\iint (2x+3)\,dx\,dy$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$



$$\int_0^{2\pi} d\varphi \int_0^1 (2r\cos\varphi + 2 + 3)\,r\,dr$$

VICE VISIĆ

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	~~$s^2F(s)-sf(0)-f'(0)$~~
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

3. $f'''(t)+f'(t)=1 \qquad x(0)=1 \quad x'(0)=1 \quad x''(0)=1$

$$s^3X(s)-s^2x(0)-sx'(0)-x''(0)+sX(s)-x(0)=\frac{1}{s}$$

$$s^3X(s)-s^2-s-1+sX(s)-1=\frac{1}{s}$$

$$s^3X(s)+sX(s)=\frac{1}{s}+s^2+s+1+1=\frac{1}{s}+s^2+s+2$$

$$X(s)(s^3+s)=\frac{1+s^3+s^2+2s}{s}=\frac{s^3+s^2+2s+1}{s}$$

$$X(s)=\frac{s^3+s^2+2s+1}{s}\cdot\frac{1}{s^3+s}=\frac{s^3+s^2+2s+1}{s(s^3+s)}=\frac{s^3+s^2+2s+1}{s(s^2+1)\cdot s}=\frac{s^3+s^2+2s+1}{s^2(s^2+1)}$$

$$\frac{s^3+s^2+2s+1}{s^2(s^2+1)}=\frac{A}{s^2}+\frac{B}{s}+\frac{Cs+D}{s^2+1}$$

$$s^3+s^2+2s+1=A(s^2+1)+Bs(s^2+1)+(Cs+D)(s^2)$$

$$s^3+s^2+2s+1=As^2+A+Bs^3+Bs+Cs^3+Ds^2$$

$$s^3+s^2+2s+1=(B+C)s^3+(A+D)s^2+(B)s+A$$

$B+C=1$ $\quad A+D=1$ $\quad A=1$ $\quad B=2$

$1+D=1$ $\quad D=0$

$B+C=1$ $\quad 2+C=1$ $\quad C=-1$

$$=A\cdot\frac{1}{s^2}+B\cdot\frac{1}{s}+\frac{Cs+D}{s^2+1}$$

$$=1\cdot t+2\cdot 1+(-1\cdot\cos(t))$$

$$=t+2-\cos(t)$$

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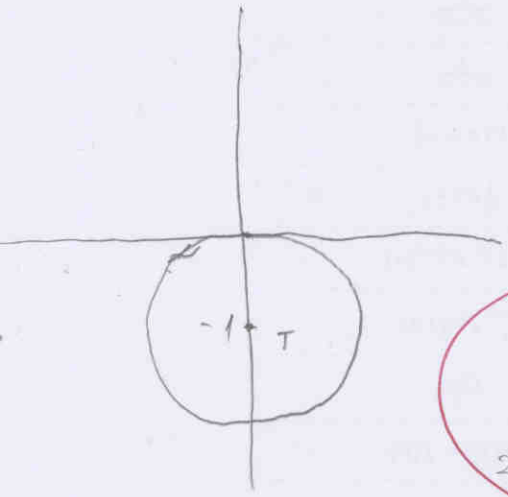
4. $r = 1$ $T(0, -1)$ $\int_{\partial K} (2x+3)\,dy$



$f \in [0, 2\pi]$

$r \in [0, 1]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$y = r\sin\varphi - 1$

$$\int_{2\pi}^{0} d\varphi \int_{0}^{1} (2r\cos\varphi + 3)\, r\, dr$$

NIJE DVOSTRUKI INTEGRAL