MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(2,1)$. Izračunati $\iint_K (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1, x'(0) = 1, x'(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izračunati $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(3)

$f'''(t) + f'(t) = 1 \qquad x(0)=1,\ x'(0)=1,\ x''(0)=1$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + sF(s) - f(0) = \frac{1}{s}$

$s^3F(s) - s^2 - s - 1 + sF(s) - 1 = \frac{1}{s}$

$F(s)(s^3+s) = \frac{1}{s} + s^2 + s + 2$

$Fs(s^3+s) = \frac{1+s^3+s^2+2s}{s}$

$F(s) = \frac{s^3+s^2+2s+1}{s(s^3+s)}$

$\frac{s^3+s^2+2s+1}{s^3(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s+1}$

$s^3+s^2+2s+1 = A(s^3+s^2) + B(s^2+s) + C(s+1) + (Ds+E)(s^3)$

$s^3+s^2+2s+1 = As^3 + As^2 + Bs^2 + Bs + Cs + C + Ds^4 + Es^3$

$0 = D \Rightarrow \boxed{D=0}$

$1 = A+E$
$1 = A+B$
$2 = B+C$
$1 = C \Rightarrow \boxed{C=1}$

$2 = B+C$
$B = 2-1$
$\boxed{B=1}$

$1 = A+B$
$A = 1-1$
$\boxed{A=0}$

$1 = A+E$
$\boxed{E=1}$

$f(t) = \frac{0}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s+1}$

$f(s) = t + \frac{1}{2}\cdot\frac{2}{s^2+1} + e^{-t}$

$f(s) = t + \frac{1}{2}t^2 + e^{-t}$ ✗

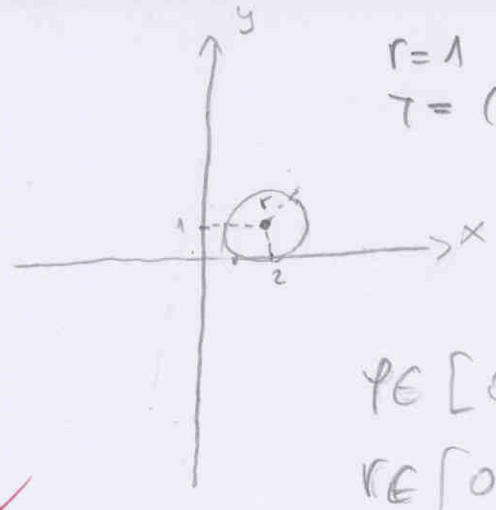
VIDI VISIC

(2.)

$$\iint_K (2x+3)\,dx\,dy$$

$$\iint_K r\cos\varphi \cdot r\sin\varphi \cdot r\,dr\,d\varphi \neq$$

$$\int_0^{2\pi} \quad \not\equiv$$



$r = 1$

$T = (2,1)$

$\varphi \in [0, 2\pi]$

$r \in [0, 1]$