MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: Matija Škiljo

BROJ INDEKSA: 54961/2009

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\,\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint_K (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1, \quad x(0) = 1, x'(0) = 1, x'(0) = 1.$$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a $\widehat{\partial K}$ kružnica orjentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1+2\cos t,\ y = 1-3\sin t,\ z = 1-3\sin t,\ t \in [0,2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(2)

$$f'''(t) + f'(t) = 1$$

$$x(0) = 1 \quad x'(0) = 1 \quad x''(0) = 1$$

$$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0) + s F(s) - f(0) = 1$$

$$s^3 F(s) - s^2 \cdot 1 - s \cdot 1 - 1 + s F(s) - 1 = \frac{1}{s^2}$$

$$s^3 F(s) - s^2 - s - 1 + s F(s) - 1 = \frac{1}{s^2}$$

$$F(s)(s^3 + s) = \frac{1}{s^2} + 1 + 1 + s + s^2$$

$$F(s)(s^3 + s) = \frac{1}{s^2} + \frac{1 + 1 + s + s^3}{1}$$

$$F(s)(s^3 + s) = \frac{1 \cdot (1 + 1 + s + s^3) s^2}{s^2} \Rightarrow 2s^2 + s^3 + s^4$$

$$F(s)(s^3 + s) = \frac{1(2s^2 + s^3 + s^4)}{s^2}$$

$$F(s)(s^3 + s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2}$$

$$F(s) = \frac{\frac{s^4 + s^3 + 2s^2 + 1}{s^2}}{\frac{s^3 + s}{1}}$$

$$F(s) = \frac{s^4 + s^3 + 2s^2 + 1}{s^2(s^3 + s)} \Rightarrow \frac{s^4 + s^3 + 2s^2 + 1}{s^2 \cdot s(s^2 + 1)}$$

$$\frac{As + B}{s^2} + \frac{C}{s} + \frac{Ds + E}{s^2 + 1} = (As + B)(s)(s^2 + 1) + C(s^2)(s^2 + 1) + (Ds + E)(s^2)(s)$$

$$(As^2 + Bs)(s^2 + 1) + Cs^3(s^2 + 1) + (Ds^3 + Es^2)s$$

$$F(s) = As^4 + As^2 + Bs^3 + Bs + Cs^4 + Cs^2 + Ds^4 + Es^3$$

$$F(s) = s^4(A + C + D) + s^3(B + E) + s^2(A + C) + s(B)$$

$$A + C + D = 1 \quad D = 0$$

$$B + E = 1 \quad 0 + E = 1 \quad \boxed{E = 1}$$

$$A + C = 2$$

$$\boxed{B = 0}$$

$$f'''(t)+f'(t)=1$$

$$x(0)=1 \quad x'(0)=1 \quad x''(0)=1$$

$$s^3F(s)-s^2f(0)-sf'(0)-f''(0)+sF(s)-f(0)=\frac{1}{s}$$

$$s^3F(s)-s^2\cdot 1-s\cdot 1-1+sF(s)-1=\frac{1}{s^2}$$

$$s^3F(s)-s^2-s+sF(s)-1=\frac{1}{s^2}$$

$$F(s)(s^3+s)=\frac{1}{s^2}+1+s+s^2$$

$$F(s)(s^3+s)=\frac{1\cdot(1+s+s^2)(s^2)}{s^2} \Rightarrow \frac{s^2+s^3+s^4+1}{s^2}$$

$$F(s)=\frac{\frac{s^4+s^3+s^2+1}{s^2}}{\frac{s^3+s}{1}}$$

$$F(s)=\frac{s^4+s^3+s^2+1}{s^2(s^3+s)} \Rightarrow \frac{s^4+s^3+s^2+1}{s^2\cdot s(s^2+1)}$$

$$\frac{As+B}{s^2}+\frac{C}{s}+\frac{Ds+E}{s^2+1}=(As+B)(s)(s^2+1)+C(s^2)(s^2+1)+(Ds+E)(s^2)(s)$$

$$(As^2+Bs)(s^2+1)+Cs^2(s^2+1)+(Ds^3+Es^2)(s)$$

$$F(s)=As^4+As^2+Bs^3+Bs+Cs^4+Cs^2+Ds^4+Es^3$$

$$s^4+s^3+s^2+1=s^4(A+C+D)+s^3(B+E)+s^2(A+C)+s(B)$$

$$A+C+D=1 \qquad 1+D=1$$

$$D=0$$

$$B+E=1 \Rightarrow E=1$$

$$A+C=1$$

$$B=0$$

?