MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	Grupa xxoxx
IME I PREZIME: MAGDA MANDIĆ **BROJ INDEKSA:** 0269015993	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. ~~20~~

2. Neka je K kugla radijusa $r = 1$ sa centrom u ishodistu. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\,\mathrm{d}S$. ~~20~~

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodistu. Izraèunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. ~~20~~

4. Neka je K krug radijusa $r = 2$ sa centrom u toèki $T(0,0)$. Izraèunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. ~~20~~

5. Neka je $\widehat{\Gamma} = \left\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$ i $\mathbf{w}(x,y,z) = (y,z,x)$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. ~~20~~

Ukupno: ~~0~~

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

(2) $r = 1$

$T(0,0)$

$\iint\limits_{\partial K} 3\,dS$

$r \in [0,1]$

$\varphi \in [0, 2\pi]$

$z \in [0, \sqrt{1-r^2}]$

$x^2 + y^2 + z^2 = 1$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 1$

$r^2(\cos^2\varphi + \sin^2\varphi) + z^2 = 1$

$r^2 \cdot 1 + z^2 = 1$

$z^2 = 1 - r^2$

$z = \sqrt{1-r^2}$

$f = 3$

TO JE SKALARNA FUNKCIJA

$F = 3$

$\operatorname{div} F = \frac{\partial 3}{\partial x} + \frac{0}{\partial y} + \frac{0}{\partial z} = 0 + 0 + 0$

$\operatorname{div} F = 0$

DIVERGENCIJA SE NE MOŽE RAČUNATI ZA SKALARNU FUNKCIJU

$\iint\limits_{\partial K} 3\,dS = \operatorname{div} F \cdot dS = \phi$

$\iint F \cdot dS = \iint \operatorname{div} F \cdot dt$

TEOREM O DIVERGENCIJI VRIJEDI ZA VEKTORSKU FUNKCIJU w

$\iiint\limits_{V} \operatorname{div} w = \iint\limits_{\delta V} (w | dS)$

1. $2x'''(t)+5x'(t)=t$

$$\left|\begin{aligned} x(0)&=1 \\ x'(0)&=0 \\ x''(0)&=0 \end{aligned}\right.$$

$$2\cdot\left(s^3X(s)-s^2\cancel{x(0)}^{1}-s\cdot\cancel{x'(0)}^{0}-\cancel{x''(0)}^{0}\right)+5\left(sX(s)-\cancel{x(0)}^{1}\right)=\frac{1}{s^2}$$

$$2\left(s^3X(s)-1s^2\right)+5\left(sX(s)-1\right)=\frac{1}{s^2}$$

$$2s^3X(s)-2s^2+5sX(s)-5=\frac{1}{s^2}$$

$$2s^3X(s)+5sX(s)=\frac{1}{s^2}+2s^2+5$$

$$(2s^3+5s)=s(2s^2+5)$$

$$X(s)(2s^3+5s)=\frac{1+2s^4+5s^2}{s^2}\;\Big/ : s(2s^2+5)$$

$$X(s)=\frac{2s^4+5s^2+1}{s^2\cdot s\cdot(2s^2+5)}=\frac{2s^4+5s^2+1}{s^3(2s^2+5)}$$

$$\frac{2s^4+5s^2+1}{s^3\cdot(2s^2+5)}=\frac{A}{s^3}+\frac{B}{s^2}+\frac{C}{s}+\frac{Ds+E}{2s^2+5}$$

$$2s^4+5s^2+1=A(2s^2+5)+B(2s^2+5)\cdot s+C(2s^2+5)\cdot s^2+(Ds+E)\cdot s^3$$

$$2s^4+5s^2+1=\underline{2As^2}+\underline{5A}+\underline{2Bs^3}+\underline{5Bs}+\underline{2Cs^4}+\underline{5Cs^2}+\underline{Ds^4}+\underline{Es^3}$$

$2=2C+D$

$0=2B+E \longrightarrow 0=2\cdot 0+E \quad \boxed{E=0}$

$5=2A+5C \longrightarrow 5=2\cdot\frac{1}{5}+5C$

$0=5B \rightarrow \boxed{B=0}$

$1=5A \Rightarrow \boxed{A=\frac{1}{5}}$

$5=\frac{2}{5}+5C \;/\cdot 5$

$25=2+25C$

$23=25C \longrightarrow \boxed{C=\frac{23}{25}}$

$2=2C+D$

$2=2\cdot\frac{23}{25}+D \;/\cdot 25$

$50=46+25D$

$4=25D \rightarrow \boxed{D=\frac{4}{25}}$

$$x(t)=\mathcal{L}^{-1}\{X(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{5}\cdot\frac{1}{s^3}+\frac{23}{25}\cdot\frac{1}{s}+\frac{\frac{4}{25}\cdot s+0}{2s^2+5}\right\}$$

$a=\sqrt{5}$

$$=\mathcal{L}^{-1}\left\{\frac{1}{5\cdot 2!}\cdot\frac{2!}{s^{2+1}}+\frac{23}{25}\cdot\frac{1}{s}+\frac{\cancel{4}^{2}}{25}\cdot\frac{s}{\cancel{2}s^2+5}\right\}$$

POGREŠNO !!! KRAĆENJE ...

$$x(t)=\frac{1}{5}\cdot t^2+\frac{23}{25}+\frac{2}{25}\cdot\cos(\sqrt{5}t)$$

$\frac{1}{10}t^2$

$\cancel{0}$

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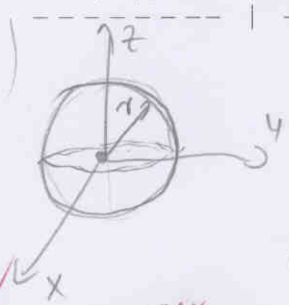
Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

$(x+y)(dx\,dy)$



(3) $r=2$

$T(0,0)$

$\iiint_K (2x+3)\,dx\,dy\,dz = ?$ ✓

$x = r\cos\varphi$

$r\in[0,2]$

$\varphi\in[0,2\pi]$

$z\in[0,\sqrt{4-r^2}]$ ✗

$x^2+y^2+z^2=r^2$ $\quad z\in[-\sqrt{4-r^2},\sqrt{4-r^2}]$

$r^2\cos^2\varphi + r^2\sin^2\varphi + z^2 = 4$

$r^2(\cos^2\varphi+\sin^2\varphi)+z^2=4$

$r^2\cdot 1 + z^2 = 4$

$z^2 = 4-r^2$

$z=\sqrt{4-r^2}$

$\iiint 2x\,dx\,dy\,dz + \iiint 3\,dx\,dy\,dz$

PORDAK MORA SLIJEDITI PORDAK

$\int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-r^2}} 2\cdot r\cos\varphi\, r\,dr\,d\varphi\,dz + \int_0^{2\pi}\int_0^{2}\int_0^{\sqrt{4-r^2}} 3r\,dr\,d\varphi\,dz =$ ✗

$2\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^2 r^2\,dr\int_0^{\sqrt{4-r^2}} dz + 3\cdot\int_0^{2\pi}d\varphi\int_0^2 r\,dr\int_0^{\sqrt{4-r^2}}dz =$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^2 r^2\,dr\,(z)\Big|_0^{\sqrt{4-r^2}} + 3\cdot\int_0^{2\pi}d\varphi\int_0^2 r\,dr\,(z)\Big|_0^{\sqrt{4-r^2}} =$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\int_0^2 r^2(\sqrt{4-r^2})\,dr + 3\cdot\int_0^{2\pi}d\varphi\int_0^2 r(\sqrt{4-r^2})\,dr =$ ✗

$\sqrt{4-r^2}=t$

$dr = dt$

$t\,dt$

$\frac{t^2}{2}$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\left[\int_0^2 r^2\,dr + \int_0^2\sqrt{4-r^2}\,dr\right] + 3\cdot\int_0^{2\pi}d\varphi\left[\int_0^2 r\,dr + \int_0^2\sqrt{4-r^2}\,dr\right] =$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\left[\left(\frac{r^3}{3}\right)\Big|_0^2 + \left(\frac{\sqrt{4-r^2}^2}{2}\right)\Big|_0^2\right] + 3\cdot\int_0^{2\pi}d\varphi\left[\left(\frac{r^2}{2}\right)\Big|_0^2 + \left(\frac{\sqrt{4-r^2}^2}{2}\right)\Big|_0^2\right] =$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\left[\frac{8}{3} + \left(\frac{4-r^2}{2}\right)\Big|_0^2\right] + 3\cdot\int_0^{2\pi}d\varphi\left[2 + \left(\frac{4-r^2}{2}\right)\Big|_0^2\right] =$

$2\cdot\int_0^{2\pi}\cos\varphi\,d\varphi\left(\frac{8}{3}\right) + 3\cdot\int_0^{2\pi}d\varphi\,(2) = 2\cdot\frac{8}{3}\cdot\sin\varphi\Big|_0^{2\pi} + 3\cdot 2\cdot\varphi\Big|_0^{2\pi} =$ →

$\Rightarrow \frac{16}{3} \cdot \left(\underset{0}{\sin 2\pi} - \underset{0}{\sin 0}\right) + 6 \cdot 2\pi = \frac{16}{3} \cdot 0 + 12\pi = 12\pi = 37{,}69$

(4) $r = 2$ $\quad r \in [0,2]$ $\quad x^2 + y^2 = r^2$

$T(0,0)$ $\quad \varphi \in [0, 2\pi]$

$\int\limits_{\partial K} (2x+3)\,ds$

$\emptyset$

(5) $x = \frac{1}{2}\cos t$

$y = \frac{1}{2}\sin t$

$z = \frac{\sqrt{3}}{2}$

$t \in [0, \pi]$

$\emptyset$