MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. IME I PREZIME: NINO MIKULANDRA BROJ INDEKSA: 57645	Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. — 20

2. Neka je K kugla radijusa $r = 1$ sa centrom u ishodistu. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\mathrm{d}S$. — 20

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodistu. Izraèunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. — 20

4. Neka je K krug radijusa $r = 2$ sa centrom u toèki $T(0,0)$. Izraèunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. — 20

5. Neka je $\widehat{\Gamma} = \left\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$ i $\mathbf{w}(x,y,z) = (y,z,x)$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. — 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

5)

$$\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2} \qquad t \in [0,\pi]$$

$$w(x,y,z) = (y,z,r)$$

$$W = \begin{bmatrix} \frac{1}{2}\cos t \\ \frac{1}{2}\sin t \\ \frac{\sqrt{3}}{2} \end{bmatrix} \checkmark$$

$$W' = \begin{bmatrix} -\frac{1}{2}\sin t \\ \frac{1}{2}\cos t \\ 0 \end{bmatrix} \checkmark$$

$$\|W'(t)\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} =$$

$$= \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} =$$

$$= \sqrt{\left[-\frac{1}{2}\sin t\right]^2 + \left[\frac{1}{2}\cos t\right]^2 + 0} =$$

NEPOTREBNO

$$= \frac{1}{4}\sin t^2 + \frac{1}{4}\cos t^2 + 0 =$$

$\Rightarrow$

$$= \frac{1}{4} \underbrace{\sin t^2 + \cos t^2}_{=1} = \frac{1}{4} \cdot 1$$

$$= \frac{1}{4}$$

$$Xor = \left(\frac{1}{2}\cos t\right)^2 + \left(\frac{1}{2}\sin t\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 =$$

$$= \frac{1}{4}\cos t^2 + \frac{1}{4}\sin t^2 + 0{,}75 =$$

$$= \frac{1}{4} \underbrace{\cos t^2 + \sin t^2}_{=1} + 0{,}75 =$$

$$= \frac{1}{4} \cdot 1 + 0{,}75 =$$

$$= 1$$

X

~~OVO JE~~

OVO NIJE

$\cancel{f}xr(r(t))$

VEKTORSKA FUNKCIJA

$$\int w \, dr = \int (Xor) \| w'(t) \| \, dt$$

POGREŠNA FORMULA

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1.) $2x'''(t) + 5x'(t) = t \quad x(0) = 1 \text{ i } x'(0) = x''(0) = 0$

$$2\mathcal{L}[x'''(t)] + 5\mathcal{L}[x'(t)] = t$$

$$2\left(s^3F(s) - s^2\overset{=1}{\cancel{x}(0)} - s\overset{=0}{\cancel{x}'(0)} - \overset{=0}{\cancel{x}''(0)}\right) + 5\left(sF(s) - \cancel{x}(0)\right) = \frac{1}{s^2}$$

$$2s^3F(s) - s^2 \cdot 1 - 0 - 0 + 5sF(s) - 1 = \frac{1}{s^2}$$

$$2s^3F(s) - s^2 + 5sF(s) - 1 = \frac{1}{s^2}$$

$$F(s)(2s^3 + 5s) = \frac{1}{s^2} + s^2 + 1 \quad / : (2s^3 + 5s)$$

$$F(s) = \frac{1 + s^2 + 1}{s^2(2s^3 + 5s)} = \frac{A}{s^2} + \frac{Bs + C}{2s^3 + 5s} \quad / \cdot s^2(2s^3 + 5s)$$

$$1 + s^2 + 1 = A(2s^3 + 5s) + Bs + C(s^2)$$

$$1 + s^2 + 1 = 2As^3 + 5As + Bs^3 + Cs^2$$

$$1 + s^2 + 1 = (A + B)s^3 + (C)s^2 + (A)s$$

$2A + B = 0$

$2 + B = 0$ $\quad \boxed{C = 0}$

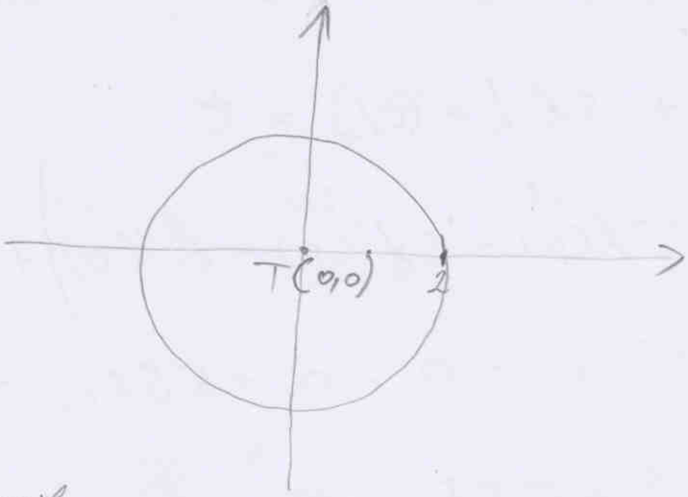
$\boxed{B = 2}$ $\quad \boxed{A = 5}$

$$\mathcal{L}^{-1} x(s) = 5\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) + \mathcal{L}^{-1}\left(\frac{2 \cdot s + 0}{2s^3 + 5s}\right)$$

$$x(s) = 5t$$

4.) $K \Rightarrow r = 2$
$T(0,0)$

$$\int_{\partial K} (2x+3)\, ds$$



$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$

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