MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: BORIS DURBIĆ

BROJ INDEKSA: 57 640

Grupa
xxoxx
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. ~~20~~
2. Neka je K kugla radijusa $r = 1$ sa centrom u ishodistu. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\,\mathrm{d}S$. 20
3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodistu. Izraèunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. ~~20~~
4. Neka je K krug radijusa $r = 2$ sa centrom u toèki $T(0,0)$. Izraèunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. ~~20~~
5. Neka je $\overset{\frown}{\Gamma} = \left\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$ i $\mathbf{w}(x,y,z) = (y,z,x)$. Izraèunati $\int\limits_{\overset{\frown}{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. ~~20~~

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1.

$$2X'''(t) + 5'(t) = t \qquad X(0)=1$$

$$X'(0) = X''(0) = 0$$

$$s^3 \cdot F(t) - s^2 \cancel{f}(0)^{1} - s\,\cancel{f'}(0)^{0} - \cancel{f''}(0)^{0}$$

$$s \cdot F(t) - \cancel{f}(0)^{1}$$

$$2\left(s^3 \cdot F(t) - s^2\right) + 5\left(s \cdot F(t) - 1\right) = \frac{1}{s^2}$$

$$2s^3 \cdot F(t) - 2s^2 + 5sF(t) - 5 = \frac{1}{s^2}$$

$$2 \cdot s^3 \cdot F(t) + 5sF(t) = \frac{1}{s^2} + 5 + 2s^2$$

$$F(t)\left(2s^3 + 5s\right) = \frac{1 + 5s^2 + 2s^4}{s^2}$$

$$F(t) = \frac{1+5s^2+2s^4}{s^2 \cdot s \cdot (2 \cdot s^2 + 5)} = \frac{1+5s^2+2s^4}{s^3(2s^2+5)} = \frac{1+5s^2+2s^4}{s^3 \cdot 2(s^2+\frac{5}{2})} = \frac{\frac{1}{2}+\frac{5}{2}s^2+s^4}{s^3 \cdot (s^2+\frac{5}{2})}$$

$$\frac{1}{2} + \frac{5}{2}s^2 + s^4 = \frac{A}{s^3} + \frac{B}{s^2} + \frac{C}{s} + \frac{Ds+E}{s^2+\frac{5}{2}}$$

$$\frac{1}{2} + \frac{5}{2}s^2 + s^4 = A\left(s^2+\frac{5}{2}\right) + Bs\left(s^2+\frac{5}{2}\right) + Cs^2\left(s^2+\frac{5}{2}\right) + (Ds+E) \cdot s^3$$

$$\frac{1}{2} + \frac{5}{2}s^2 + s^4 = \cancel{A}s^2 + \cancel{\frac{5}{2}}A + \cancel{B}s^3 + \cancel{\frac{5}{2}}Bs + \cancel{C}s^4 + \cancel{\frac{5}{2}}Cs^2 + \cancel{D}s^4 + \cancel{E}s^3$$

NASTAVAK

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s^4 $1 = C + D$

s^3 $0 = B + E$

s^2 $\frac{5}{2} = A + \frac{5}{2} C$

s $0 = \frac{5}{2} B$

$\frac{1}{2} = \frac{5}{2} A$

$A = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} = \frac{1}{5}$ ✓

$B = 0$

$E = 0$

$\frac{5}{2} = \frac{1}{5} + \frac{5}{2} c$

$\frac{5}{2} c = \frac{5}{2} - \frac{1}{5} = \frac{25 - 2}{10}$

$\frac{5}{2} c = \frac{23}{10}$

$c = \frac{23}{25}$

$\frac{23}{\cancel{10}_5} \cdot \frac{\cancel{2}}{5} = \frac{23}{25}$

$1 = \frac{23}{25} + D$

$D = \frac{2}{25}$

✓

$$\frac{\frac{1}{5}}{s^3} + \frac{0}{s^2} + \frac{\frac{23}{25}}{s} + \frac{\frac{2}{25} s + 0}{s^2 + \frac{5}{2}} \Rightarrow \frac{1}{5} \cdot \frac{1}{6} \cdot t^2 + \frac{23}{25} + \frac{2}{25} \frac{s}{s^2 - \sqrt{\frac{5}{2}}^2}$$

×

× ✓ ✓

$$\Rightarrow \frac{1}{30} t^2 + \frac{23}{25} + \frac{2}{25} \cdot \cos\left(\sqrt{\frac{5}{2}} t\right)$$

× 0

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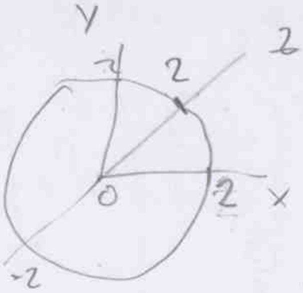
3. $r=2$ $\iiint\limits_{K}(2x+3)\,dx\,dy\,dz$



$x = r\cos\varphi$

0

$$\int_0^{2\pi} d\varphi \int_{-2}^{2} dz \int_0^{\textcircled{2}} (2r\cos\varphi + 3)\,r\,dr \quad \times \quad \times$$

$$\int_0^{2\pi} d\varphi \int_{-2}^{2} dz \int_0^{2} 2r^2\cos\varphi + 3r = \int_0^{2\pi} d\varphi \int_{-2}^{2}\left(2\cdot\frac{r^3}{3}\cdot\cos\varphi + 3\frac{r^2}{2}\right]_0^2 = \int_0^{2\pi} d\varphi \int_{-2}^{2}\left(\frac{16}{3}\cos\varphi + 6\right)dz$$

$$= \int_0^{2\pi} d\varphi \left(\frac{16}{3}\cos\varphi\, z + 6z\right]_{-2}^{2} = \int_0^{2\pi}\left(\frac{16}{3}\cdot 2\cos\varphi + 6\cdot 2 - \left(-2\cdot\frac{16}{3}\cos\varphi - 2\cdot 6\right)\right.$$

$$= \int_0^{2\pi}\left(\frac{64}{3}\cos\varphi + 24\right)d\varphi = \cancel{\frac{64}{3}\cdot\cos 2\pi} + 48\pi - \cancel{\frac{64}{3}\cdot\cos 0} = 48\pi$$

5. $\tilde{r}\ \left\{ (x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi] \right\}\ v(x,y,z) = (y,z,x)$

$$r = \begin{bmatrix} \frac{1}{2}\cos t \\ \frac{1}{2}\sin t \\ \frac{\sqrt{3}}{2} \end{bmatrix} \checkmark \quad r' = \begin{bmatrix} -\frac{1}{2}\sin t \\ \frac{1}{2}\cos t \\ 0 \end{bmatrix} \checkmark$$

$$|r'| = \sqrt{\left(-\frac{1}{2}\sin t\right)^2 + \left(\frac{1}{2}\cos t\right)^2 + 0}$$

$$|r'| = \sqrt{\frac{1}{4}\sin^2 t + \frac{1}{4}\cos^2 t}$$

$$|r'| = \sqrt{\frac{1}{4}(\sin^2 t + \cos^2 t)} = \sqrt{\frac{1}{4}} = \frac{1}{16}$$

$$\int_0^{\pi} \left(\sin t + \frac{\sqrt{3}}{2} + \cos t\right) \frac{1}{16}\, dt$$ ✗

4. $r = 2 \quad T(0,0) \quad \int_{\partial K} (2x+3)\, ds \quad r = \cos\varphi$

$$\int_0^{2\pi} d\varphi \int_0^{2} (2\cos\varphi + 3)\, dr$$ ← POGREŠAN TIP INTEGRALA