MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. 20

2. Neka je K kugla radijusa $r = 1$ sa centrom u ishodistu. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\mathrm{d}S$. 20

3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodistu. Izraèunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20

4. Neka je K krug radijusa $r = 2$ sa centrom u toèki $T(0,0)$. Izraèunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20

5. Neka je $\widehat{\Gamma} = \left\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$ i $\mathbf{w}(x,y,z) = (y,z,x)$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: 20

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

4.

$$\int_{\partial K} (2x+3)\,ds = \begin{bmatrix} \text{KRUG} \\ r=2 \\ T(0,0) \end{bmatrix} =$$

NISTE PREPOZNALI DA JE RIJEČ O KRIVULJNOM INTEGRALU

$$\int_0^{2\pi}\int_0^2 (2r\cos\varphi+3)\,r\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^2 (2r^2\cos\varphi+3r)\,dr\,d\varphi =$$

$$= \int_0^{2\pi}\left(2\cdot\frac{r^3}{3}\cos\varphi\Big|_0^2 + 3\cdot\frac{r^2}{2}\Big|_0^2\right)d\varphi =$$

$$= \int_0^{2\pi}\left(\frac{16}{3}\cos\varphi+6\right)d\varphi = \frac{16}{3}\sin\varphi\Big|_0^{2\pi} + 6\varphi\Big|_0^{2\pi} =$$

$$= \frac{16}{3}(\sin 2\pi - \sin 0) + 6\cdot(2\pi-0) = 12\pi$$

3.

$$\iiint_K (2x+3)\,dx\,dy\,dz = \begin{bmatrix} \text{sferne kordinate} \\ x = r\sin\theta\cos\varphi \end{bmatrix} =$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^2 (2r\sin\theta\cos\varphi+3)\cdot r^2\sin\theta\,dr\,d\theta\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(2\sin^2\theta\cos\varphi\cdot\frac{r^3}{3} + 3\frac{r^3}{3}\sin\theta\right)\Big|_0^2 d\theta\,d\varphi =$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{16}{3}\sin^2\theta\cos\varphi + 8\sin\theta\right)d\theta\,d\varphi =$$

$\longrightarrow$

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(3.)

$$= \int_0^{2\pi} \left(\frac{16}{3} \cos\varphi \int_0^{\pi} \sin^2\theta \, d\theta + 8 \int_0^{\pi} \sin\theta \, d(\theta) \right) d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{16}{3} \cos\varphi \left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \right) \Big|_0^{\pi} + 8(-\cos\theta) \Big|_0^{\pi} \right) d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{16}{3} \cos\varphi \, \frac{1}{2}\pi + 8(-\cos\pi + \cos 0) \right) d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{8\pi}{3} \cos\varphi + 8(-(-1)+1) \right) d\varphi =$$

$$= \int_0^{2\pi} \left(\frac{8\pi}{3} \cos\varphi + 16 \right) d\varphi = \frac{8\pi}{3} \sin\varphi \Big|_0^{2\pi} + 16\varphi \Big|_0^{2\pi}$$

$$= 16 \cdot 2\pi = 32\pi$$ ✓ 20

(2.) $\int_{\partial K} 3 \, ds$: [KUGLA r = 1] =

NISTE PREPOZNALI DA JE RIJEČ O PLOŠNOM INTEGRALU

$$= \int_0^{2\pi} \int_0^{\pi} \int_0^{1} 3r^2 \sin\theta \, dr \, d\theta \, d\varphi =$$ X

$$= \int_0^{2\pi} \int_0^{\pi} \cancel{3} \sin\theta \frac{r^3}{\cancel{3}} \Big|_0^1 d\theta \, d\varphi = \int_0^{2\pi} \int_0^{\pi} \sin\theta \, d\theta \, d\varphi =$$

$$= \int_0^{2\pi} -\cos\theta \Big|_0^{\pi} d\varphi = \int_0^{2\pi} (-\cos\pi + \cos 0) \, d\varphi =$$

$$= \int_0^{2\pi} 2 \, d\varphi = 2\varphi \Big|_0^{2\pi} = 4\pi$$

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