MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	Grupa xxoxx
IME I PREZIME: GREGOR HAMARIĆ BROJ INDEKSA: 57650	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $2x'''(t) + 5x'(t) = t$, $x(0) = 1$ i $x'(0) = x''(0) = 0$. 20
2. Neka je K kugla radijusa $r = 1$ sa centrom u ishodistu. Preko definicije plošnog integrala izračunati $\iint\limits_{\partial K} 3\,\mathrm{d}S$. 20
3. Neka je K kugla radijusa $r = 2$ sa centrom u ishodistu. Izraèunati $\iiint\limits_{K} (2x+3)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z$. 20
4. Neka je K krug radijusa $r = 2$ sa centrom u toèki $T(0,0)$. Izraèunati $\int\limits_{\partial K} (2x+3)\,\mathrm{d}s$. 20
5. Neka je $\widehat{\Gamma} = \left\{(x,y,z) : x = \frac{1}{2}\cos t,\ y = \frac{1}{2}\sin t,\ z = \frac{\sqrt{3}}{2},\ t \in [0,\pi]\right\}$ i $\mathbf{w}(x,y,z) = (y,z,x)$. Izraèunati $\int\limits_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno: ∅

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $2x'''(t) + 5x'(t) = t \quad , \quad x(0) = 1 \quad , \quad x'(0) = x''(0) = 0$

$2\cdot\left(s^3F(s) - s^2 f(0) - s\cancel{f'(0)} - \cancel{f''(0)}\right) + 5\cdot\left(sF(s) - f(0)\right) = \frac{1}{s^2}$

$2\cdot\left(s^3X(s) - s^2x(0) - s\cancel{x'(0)} - \cancel{x''(0)}\right) + 5\cdot\left(sX(s) - X(0)\right) = \frac{1}{s^2}$

$2s^3X(s) - 2s^2 + 5sX(s) - 5 = \frac{1}{s^2}$

$s\left(2s^2X(s) + 5X(s)\right) = \frac{1}{s^2} + 2s^2 + 5$

$X(s)\cdot s\left(2s^2 + 5\right) = \frac{1 + 2s^4 + 5s^2}{s^2}$

2 $X(s) = \frac{s^4 + 5s^2 + 1}{s\cdot s^2\cdot\left(s^2 + \frac{5}{2}\right)}$

$\frac{s^4 + 5s^2 + 1}{s\cdot s^2\cdot\left(s^2 + \frac{5}{2}\right)} = \frac{A}{s} + \frac{B}{s} + \frac{C}{s^2} + \frac{Ds + E}{s^2 + \frac{5}{2}}$

1. $2x'''(t) + 5x'(t) = t$ $\quad x(0)=1, \ x'=x''=0$ GREGOR HAMARIĆ

$$2\cdot\left(s^3X(s) - s^2 x(0) - \cancel{s x'(0)} - \cancel{x''(0)}\right) + 5\cdot\left(sX(s) - x(0)\right) = \frac{1}{s^2}$$

$$2s^3X(s) - 2s^2 + 5sX(s) - 5 = \frac{1}{s^2}$$

$$2s^3X(s) + 5sX(s) = \frac{1}{s^2} + 2s^2 + 5$$

$$2s^3X(s) + 5sX(s) = \frac{1+2s^4+5s^2}{s^2}$$

X

$$X(s)\left(2s^3+5s\right) = \frac{s^4+5s^2+1}{s^2}$$

$$X(s)\,2\left(s^3+\frac{5}{2}s\right) = \frac{s^4+5s^2+1}{s^2}$$

$$X(s) = \frac{\frac{1}{2}s^4+\frac{5}{2}s^2+\frac{1}{2}}{s\cdot s^2\cdot s^3\cdot\left(s^2+\frac{5}{2}\right)}$$

$$\frac{\frac{1}{2}s^4+\frac{5}{2}s^2+\frac{1}{2}}{s^3\cdot\left(s^2+\frac{5}{2}\right)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{Ds+E}{s^2+\frac{5}{2}} \quad / \cdot s^3\cdot\left(s^2+\frac{5}{2}\right)$$

$$\frac{1}{2}s^4+\frac{5}{2}s^2+\frac{1}{2} = As^2\left(s^2+\frac{5}{2}\right) + Bs\left(s^2+\frac{5}{2}\right) + C\left(s^2+\frac{5}{2}\right) + (Ds+E)s^3$$

$$\frac{1}{2}s^4+\frac{5}{2}s^2+\frac{1}{2} = As^4 + \frac{5}{2}As^2 + Bs^3 + \frac{5}{2}Bs + Cs^2 + \frac{5}{2}C + Ds^4 + Es^3$$

$$\frac{1}{2}s^4+\frac{5}{2}s^2+\frac{1}{2} = (A+D)s^4 + (B+E)s^3 + \left(\frac{5}{2}A+C\right)s^2 + \left(\frac{5}{2}B\right)s + \frac{5}{2}C$$

$A+D=\frac{1}{2}$

$B+E=0$

$\frac{5}{2}A+C=\frac{5}{2}$

$\frac{5}{2}B=0 \quad \boxed{B=0}$

$\frac{5}{2}C=\frac{1}{2} \quad \boxed{E=0}$

$5C=1$

$\boxed{C=\frac{1}{5}}$ ✓

$\frac{5}{2}A+\frac{1}{5}=\frac{5}{2}$

$25A+2=25$

$25A=23$

$\boxed{A=\frac{23}{25}}$ ✓

$\frac{23}{25}+D=\frac{1}{2}$

$46+50D=25$

$50D=25-46$

$50D=-21$

$\boxed{D=-\frac{21}{50}}$

$\frac{23}{25s}+\frac{1}{5s^3}+\frac{21}{(50s^2+125)}$

NISTE NITI POKUŠALI PROVJERITI $f(0)=1$

$$X(s) = \frac{23}{25}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{1}{s^3} - \frac{21}{50}\cdot\frac{s}{s^2+\frac{5}{2}}$$

X

$\frac{1}{5}\cdot\frac{1}{s^3} = \frac{1}{10}\cdot\frac{2}{s^3}$

$$f(t) = \frac{23}{25} + \frac{1}{10}t^2 - \frac{21}{50}\cos\sqrt{\frac{5}{2}}t$$

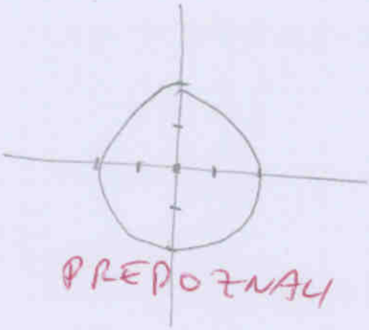
GREGOR HAMARIĆ

4. $T(0,0)$

$r = 2$

$r \in [0,2]$

$\varphi \in [0,2\pi]$



$$\int_{\partial K} (2x+3)\, ds$$

$$\int_0^{2\pi} \int_0^2 (2x+3)\, dx\, dy =$$

NISTE PREPOZNALI O KOJOJ VRSTI INTEGRALA JE RIJEČ.

$$= 2\pi \int_0^2 (2r\cos\varphi + 3)\, r\, dr =$$

$$= 2\pi \int_0^2 (2r^2 \cdot 1 + 3r)\, dr =$$

$$= 2\pi \int_0^2 (2r^2 + 3r)\, dr =$$

$$= 2\pi \; 2 \cdot 2^2 + 3 \cdot 2 =$$

$$= 2\pi \; 8 + 6 = 2\pi \; 14$$

$$= 28\pi$$