| **MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. | Grupa xxoxo |
|---|---|
| IME I PREZIME: RJEŠENJE 3     BROJ INDEKSA: | POPUNJAVA NASTAVNIK Broj ↓ bodova |

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0, \; f(0) = f''(0) = 2.$$

2. Neka je $K$ kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka $C$ plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru $z$ osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint_{\cancel{\partial K}\; C} 2x + 3\mathrm{d}y\mathrm{d}z$. 20

4. Zadana je krivulja $C$ s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$ , $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int_C f\,\mathrm{d}s$ 20

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int_{\widehat{\Gamma}} \frac{x\mathrm{d}x + y\mathrm{d}y}{\sqrt{3 + x^2 + y^2}} =$$

20

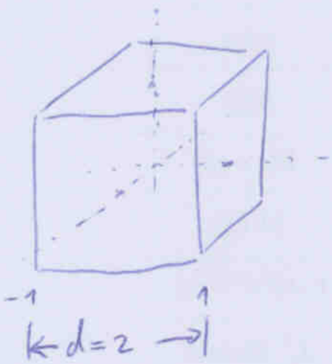
Ukupno:

Tablica integrala

| | | |
|---|---|---|
| $\int \mathrm{d}x = x + C$ | $\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$ | $\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$ |
| $\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\; n \neq -1$ | $\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$ | $\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$ |
| $\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$ | $\int \sinh x \,\mathrm{d}x = \cosh x + C$ | $\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$ |
| $\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$ | $\int \cosh x \,\mathrm{d}x = \sinh x + C$ | $\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$ |
| $\int \sin x \,\mathrm{d}x = -\cos x + C$ | $\int \tanh x \,\mathrm{d}x = \ln\lvert\cosh x\rvert$ | $\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$ |
| $\int \cos x \,\mathrm{d}x = \sin x + C$ | $\int \coth x \,\mathrm{d}x = \ln\lvert\sinh x\rvert$ | $\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$ |
| $\int \tan x \,\mathrm{d}x = -\ln\lvert\cos x\rvert$ | $\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$ | $\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$ |
| $\int \cot x \,\mathrm{d}x = \ln\lvert\sin x\rvert$ | $\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$ | $\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$ |

1) VIDI PEZEROVIĆ

② K ... kocka, stranica $a=2$, centar u $T(0,0,0)$



$$\iint_{\partial K} (2x+3)\,\underbrace{dx\,dy}\; = (*) \qquad \Rightarrow \quad w=\begin{pmatrix}0\\0\\2x+3\end{pmatrix}$$

PLOŠNI INTEGRAL

U TERMINIMA PLOŠNOG INTEGRALA OZNAČAVA 2. VRSTU

ORIJENTACIJA NIJE SPECIFICIRANA ... UZMIMO VANJSKU ORIJENTACIJU

PO TEOREMU O DIVERGENCIJI

$$(*) = \iint_{\partial K} (w \mid dS) = \iiint_K \operatorname{div} w = 0 \qquad \text{ZBOG } \operatorname{div} w = 0$$

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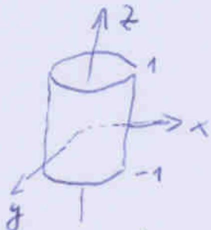
③ C plašt cilindra, $r=1$



PARAMETRIZACIJA

$r(u,v)=(\cos u, \sin u, v)$

$u \in [0, 2\pi]$

$v \in [-1, 1]$

$$\iint_C (2x+3)\,dy\,dz = \left\{ \begin{matrix} \text{PLOŠNI} \\ \text{INTEGRAL} \\ \text{2. VRSTE} \end{matrix} \right\} \qquad w=\begin{pmatrix}2x+3\\0\\0\end{pmatrix} \qquad \frac{\partial r}{\partial u}=\begin{pmatrix}-\sin u\\ \cos u\\0\end{pmatrix} \quad \frac{\partial r}{\partial v}=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\vec{n} = \frac{\partial r}{\partial u}\times\frac{\partial r}{\partial v} = \begin{pmatrix}\cos u\\ \sin u\\ 0\end{pmatrix}$$

$$= \int_0^{2\pi}\int_{-1}^{1} \begin{pmatrix}2\cos u+3\\0\\0\end{pmatrix}\cdot\begin{pmatrix}\cos u\\ \sin u\\0\end{pmatrix} dv\,du = \int_0^{2\pi}\int_{-1}^{1} 2\cos^2 u + 3\cos u \,dv\,du =$$

$$= 2\Bigg[2\underbrace{\int_0^{2\pi}\cos^2 u\,du}_{=\int_0^{2\pi}\frac{1+\cos(2u)}{2}du} + 3\underbrace{\int_0^{2\pi}\cos u\,du}_{=0}\Bigg] = 2\cdot 2\cdot\frac{1}{2}\cdot 2\pi = 4\pi$$

④ $r(t) = \begin{pmatrix} \cos t + 1 \\ \frac{t}{2} \\ \sin t \end{pmatrix}$ $r'(t) = \begin{pmatrix} -\sin t \\ \frac{1}{2} \\ \cos t \end{pmatrix}$ $\|r'(t)\| = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$

$$\int_C f\, ds = \left\{ \begin{matrix} \text{KRIVULJNI} \\ \text{INTEGRAL} \\ \text{1. VRSTE} \end{matrix} \right\} = \int_0^{4\pi} \Big[ \underbrace{(\cos t + 1)^2 + \left(\tfrac{t}{2}\right)^2 + (\sin t)^2}_{\underbrace{\cos^2 t + 2\cos t + 1 + \frac{t^2}{4} + \sin^2 t}_{= 1 + 1 + 2\cos t + \frac{t^2}{4}}} \Big] \cdot \frac{\sqrt{5}}{2}\, dt$$

$$= \frac{\sqrt{5}}{2} \int_0^{4\pi} 2 + 2\cos t + \frac{t^2}{4}\, dt = \frac{\sqrt{5}}{2} \Bigg[ \underbrace{\int_0^{4\pi} 2\, dt}_{= 8\pi} + 2 \underbrace{\int_0^{4\pi} \cos t\, dt}_{= 0} + \underbrace{\int_0^{4\pi} \frac{t^2}{4}\, dt}_{\frac{1}{12}(4\pi)^3} \Bigg] = \frac{\sqrt{5}}{2}\left(8\pi + \frac{16}{3}\pi^3\right)$$

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⑤ VIDI SEMINAR "KRIVULJNI INTEGRAL DRUGE VRSTE"

PRIMJER OBRAĐEN NA SEMINARU.