MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,dxdy$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint\limits_{\partial K} 2x + 3dydz$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int\limits_C f\,ds$ 20

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int\limits_{\widehat{\Gamma}} \frac{xdx + ydy}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno: 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1. $2(s^3F(s) - s^2f(0) - sf'(0) - f''(0)) + 2(s^2F(s) - sf(0) - f'(0)) = 0$

$2(s^3F(s) - 2s^2 - 2) + 2(s^2F(s) - 2s) = 0$

$2s^3F(s) - 4s^2 - 4 + 2s^2F(s) - 4s = 0$

$F(s)(2s^3 + 2s^2) = 4s^2 + 4s + 4$

$F(s) = \frac{4s^2 + 4s + 4}{s^2(2s+2)}$ ✓

$\frac{4s^2 + 4s + 4}{s^2(2s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(2s+2)}$

$4s^2+4s+4 = As(2s+2)+B(2s+2)+Cs^2$

$4s^2+4s+4 = 2As^2+2As+2Bs+2B+Cs^2$

$4s^2+4s+4 = (2A+C)s^2+(2A+2B)s+2B$

$2A+C=4$ $\quad B=\frac{4}{2}=2$ $\quad 2A+C=4$

$2A+2B=4$ $\quad 2A+4=4$ $\quad C=4$

$2B=4$ $\quad 2A=0$

$A=0$

$$F(s)=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{2s+2}=\frac{0}{s}+\frac{2}{s^2}+\frac{4}{2s+2}=2\cdot\frac{1}{s^2}+\frac{4}{2(s+1)}$$

$$F(s)=2\cdot\frac{1}{s^2}+\frac{4}{2}\cdot\frac{1}{s+1}=2t+\frac{4}{2}e^{-t}$$

$F(s)=2t+2e^{-t}$ ✓ 20