MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

IME I PREZIME: BRUNO LIPOTILA **BROJ INDEKSA:** 54960

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint\limits_{\partial K} 2x + 3\mathrm{d}y\mathrm{d}z$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int\limits_C f\,\mathrm{d}s$ 20

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int\limits_{\widehat{\Gamma}} \frac{x\mathrm{d}x + y\mathrm{d}y}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln \lvert \cosh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln \lvert \sinh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln \lvert \cos x \rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln \lvert \sin x \rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

(1) $2f'''(s) + 2f''(s) = 0$ $\quad f'(0) = 0$

$f(0) = f''(0) = 2$

$2\left(s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)\right) + 2\left(s^2 F(s) - s f(0) - f'(0)\right)$

$= 0$

$2(s^3F(s) - s^2 \cdot 2 - 2) + 2(s^2F(s) - s2)$

$2s^3F(s) - 2s^2 - 2 + 2s^2F(s) - 2s$

VUČE GREŠKU PACIJE...

$2s^3F(s) + 2s^2F(s) = 2s^2 + 2 + 2s$

$F(s)(2s^3 + 2s^2) = 2s^2 + 2 + 2s$

$F(s)\, s^2(2s+2) = 2s^2 + 2 + 2s \;/\; s^2(2s+2)$

$$\frac{2s^2+2+2s}{s^2(2s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{2s+2}$$

$A(s)(2s+2) + B(2s+2) + Cs + D(s^2)$

$2As^2 + 2A + 2Bs + 2B + Cs^2 + Ds^2$

$2 = A + C + D$

$2 = 2B \quad B = 0$

$2 = 2A + 2B$

$B = 2 - 2 = 0$

$2 = 1 + C + D$

$2 =$

$2 = 2A + 0$

$2A = 2 - 2$

$2A = 2$

$A = \frac{2}{2} = 1$

$D = -2$

$C = 1$

LIPOTICA

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\, f(t)$	$(-1)^n\, F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

NASTAVAK (1) ZADATKA

$f(t) = \frac{1}{s} + s^2 + \frac{s-2}{2s+2}$

$f(t) = 1 + t + \cos(2t)$