MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Grupa xxoxo

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint\limits_{\partial K} 2x + 3\mathrm{d}y\mathrm{d}z$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int\limits_C f\,\mathrm{d}s$ 20

5. Neka je $\overset{\frown}{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int\limits_{\overset{\frown}{\Gamma}} \frac{x\mathrm{d}x + y\mathrm{d}y}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln \lvert \cosh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln \lvert \sinh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln \lvert \cos x \rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln \lvert \sin x \rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

1) $2f'''(t) + 2f''(t) = 0$

$f(0) = 0$ ITALICAS 57661

$f(0) = 0$ 2

$f''(0) = 2$

$$2(s^3F(s) - s^2\cancel{f(0)}^{0} - s\cancel{f'(0)}^{0} - f''(0)^{2}) + 2(s^2F(s) - s\cancel{f(0)}^{0} - f'(0)^{0}) = 0$$

$$2s^3F(s) + 2s^2F(s) = 2$$

$$F(s)(2s^3 + 2s^2) = 2$$

$$F(s)\,2s^2(s+1) = 2$$

$$F(s) = \frac{2}{2s^2(s+1)}$$

$$\frac{2}{2s^2+(s+1)} = \frac{A}{2s^2} + \frac{B}{2s} + \frac{Cs+D}{s+1}$$

$$= A(s+1) + Bs(s+1) + (Cs+D)\,2s^2$$

$$= As + A + Bs^2 + Bs + 2Cs^3 + 2Ds^2$$

$0 = 2C \rightarrow \boxed{C=0}$

$0 = B + 2D \longrightarrow 0 = -2 + 2D$

$0 = A + B \rightarrow 0 = 2 + B \qquad 2 = 2D \mid \cdot\frac{1}{2}$

$\boxed{2 = A} \qquad \boxed{B = -2} \qquad \boxed{D = 1}$

$$= \frac{2\cancel{1}}{\cancel{2}s^2} - \frac{\cancel{2}1}{\cancel{2}s} + \frac{1}{s+1}$$

$$= t - 1 + \sin(t)$$