MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.	Grupa xxoxo
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1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = 2f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint_{\partial K} 2x + 3\mathrm{d}y\mathrm{d}z$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izračunati $\int_C f\,\mathrm{d}s$ 20

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\dfrac{x^2}{3} + \dfrac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int_{\widehat{\Gamma}} \frac{x\mathrm{d}x + y\mathrm{d}y}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

1. $2s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + 2s^2F(s) - s f(0) - f'(0) = 0$

$2(s^3F(s) - s^2\cdot 2 - s\cdot 0 - 2) + 2(s^2F(s) - 2s - 0) = 0$

$2(s^3F(s) - 2s^2 + 2) + 2(s^2F(s) - 2s) = 0$

$2s^3F(s) - 4s^2 + 4 + 2s^2F(s) - 4s = 0$

$F(s)(2s^3 + 2s^2) - 4s^2 + 4 - 4s = 0$

$F(s)(2s^3 + 2s^2) = 4s^2 - 4 + 4s$

$F(s) = \dfrac{4s^2 - 4 + 4s}{2s^3 + 2s^2}$ ✗

→

$$F(s) = \frac{4s^2-4+4s}{2s^3+2s^2} = \frac{4s^2-4+4s}{2s^2(s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}$$

$$4s^2-4+4s = A\,2(s+1) + B\,2s(s+1) + C\,2s^2$$

$$4s^2-4+4s = A(2s+2) + B(2s^2+2s) + 2Cs^2$$

$$= 2As + 2A + 2Bs^2 + 2Bs + 2Cs^2$$

$$2B + 2C = 4$$

$$2A + 2B = 4$$

$$2A = -4 \qquad 2\cdot(-2) + 2B = 4 \qquad 2B + 2C = 4$$

$$A = -\frac{4}{2} \qquad -4 + 2B = 4 \qquad 2\cdot 4 + 2C = 4$$

$$A = -2 \qquad 2B = 4+4 \qquad 8 + 2C = 4$$

$$2B = 8 \qquad 2C = 4-8$$

$$B = \frac{8}{2} \qquad 2C = -4$$

$$B = 4 \qquad C = -\frac{4}{2}$$

$$C = -2$$

$$\frac{-2}{s^2} + \frac{4}{s} + \frac{-2}{s+1} = -2\cdot\frac{1}{s^2} + 4\cdot\frac{1}{s} - 2\cdot\frac{1}{s+1}$$

$$f(t) = -2\cdot t + 4\cdot 1 - 2\cdot e^{-t}$$

$$f(t) = -2t + 4 - 2e^{-t}$$

ZAŠTO NEMA PROVJERE?

2.

$a \in [0,2]$

$$\int_0^2 \int_0^2 (2x+3)\,dx\,dy$$

$$\int_0^2 2xy+3y \Big/_0^2 dx = \int_0^2 (2xy+3y)\Big/_0^2 dx$$

$$= \int_0^2 2x \cdot 2 + 3 \cdot 2 \quad dx$$

$$= \int_0^2 4x+6 \quad dx$$

$$= \cancel{4}^2 \frac{x^2}{\cancel{2}} + 6x \Big/_0^2$$

$$= 2x^2 + 6x \Big/_0^2$$

$$= 2 \cdot 2^2 + 6 \cdot 2$$

$$= 2 \cdot 4 + 12$$

$$= 20$$

TOMISLAV ZRILIĆ

4.

$t \in [0, 4\pi]$

$x = \cos(t) + 1$

$y = \frac{t}{2}$

$z = \sin t$

$$\int_C \rho \, ds = \int_0^{4\pi} (x^2 + y^2 + z^2) \cdot \sqrt{x'^2 + y'^2 + z'^2}$$

$y = 1$

$z = \cos t$

$x' = -\sin(t) + 1$

$$\int_C \rho \, ds = \int_0^{4\pi} (\cos(t)+1)^2 + \left(\frac{t}{2}\right)^2 + (\sin t)^2 \cdot \sqrt{(-\sin(t)+1)^2 + 1^2 + (\cos(t))^2} \, dt$$

$\times$

$$\int_0^{4\pi} \cos^2 t + 1 + \frac{t^2}{2} + \sin^2 t \cdot \sqrt{\sin^2 t + 1 + 1 + \cos^2 t} \, dt$$

$\varnothing$

$$\int_0^{4\pi} \cos^2 t + \sin^2 t + 1 + \frac{t^2}{2} \sqrt{\sin^2 t + \cos^2 t + 2} \, dt$$

$$\int_0^{4\pi} 2 + \frac{t^2}{2} \sqrt{3} \, dt$$

$$\int_0^{4\pi} 2 + t^2 \cdot \frac{1}{2} \sqrt{3} \, dt$$

$$2t + \frac{t^3}{3} \cdot \frac{1}{2} t \sqrt{3} \, t \Big/_0^{4\pi}$$

$$2 \cdot 4\pi + \frac{(4\pi)^3}{3} + \frac{1}{2} 4\pi \sqrt{3} \cdot 4\pi$$

$$8\pi + \frac{64}{3} + 2\pi \cdot 4\sqrt{3}\pi$$

$$10\pi + \frac{64}{3} 4\sqrt{3}\pi$$

TOMISLAV ZRILIĆ

3\. $r \in [0,1]$

$v \in [0,2]$

$z \in [-1,1]$

$$\int_0^2 \int_0^1 \int_{-1}^1 2x + 3 dy\, dz$$

?

5\. $\frac{x^2}{3} + \frac{y^2}{15} = 1$

$$\int_{\Gamma} \frac{x\,dx + y\,dy}{\sqrt{3 + x^2 + y^2}} =$$

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