MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint\limits_{\partial K} 2x + 3\mathrm{d}y\mathrm{d}z$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int\limits_C f\,\mathrm{d}s$ 20

5. Neka je $\overset{\frown}{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int\limits_{\overset{\frown}{\Gamma}} \frac{x\mathrm{d}x + y\mathrm{d}y}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln \lvert \cosh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln \lvert \sinh x \rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln \lvert \cos x \rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln \lvert \sin x \rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

$2f'''(t)+2f''(t)=0 \quad f'(0)=0 \quad f(0)=f''(0)=2$ Mateja Mitrović

$2\cdot\left(s^3F(s)-s^2\underbrace{f(0)}_{2}-s\underbrace{f'(0)}_{0}-\underbrace{f''(0)}_{2}\right)+2\left(s^2F(s)-s\underbrace{f(0)}_{2}-\underbrace{f'(0)}_{0}\right)=0$

$2s^3F(s)-2s^2-2+2s^2F(s)-2s=0$

$F(s)\,2s^3+2s^2=2s^2+2+2s \mid :2s^3+2s^2$

$F(s)=\frac{2s^2+2+2s}{2s^3+2s^2}=\frac{2s^2+2+2s}{2s^2\cdot(s)+2s(s)}$

$F(s)=\frac{s^2+2+s}{s^2(s)+s(s)} \mid s^2(s)+s(s)$

$s^2+2+s=\frac{A}{s^2(s)}+\frac{B}{s(s)}$

$s^2+2+s=As(s)+Bs^2(s)$

$s^2+2+s=As^2+Bs^3$

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$r\begin{pmatrix}x\\y\\z\end{pmatrix}$ $r'(t)=\begin{pmatrix}\sin t\\ \frac{1}{2}\\ -\cos t\end{pmatrix}$ $f(x,y,z)=(\sin t)^2+\left(\frac{1}{2}\right)^2+(-\cos t)^2$

$f(x,y,z)=\sin t^2+\frac{1}{4}+\cos t^2=\sin t^2+\cos t^2+\frac{1}{4}=$

$f(x,y,z)=\frac{5}{4}$

$$\int_0^{4\pi} f\,ds=\int_0^{4\pi}\frac{5}{4}\,ds=\frac{5}{4}\int_0^{4\pi}ds=\frac{5}{4}s\Big|_0^{4\pi}=\frac{5}{4}\cdot 4\pi=5\pi$$

2. $$\int_0^1\int_0^2 \frac{x^2}{\partial y}+\frac{3y}{\partial x}=\int_0^1 \frac{x^2}{\partial y}+y\Big|_0^1=\int_0^1 \frac{x^2}{\partial y}+1=\frac{1}{2}\Big|_0^1+1=\frac{1}{2}+1=\frac{3}{2}$$

3. $$\int_0^1\int_{-1}^1 \frac{2x}{\partial y}+\frac{3y}{\partial z}+\frac{z}{\partial x}=\int_0^1 \frac{2x}{\partial y}+\frac{3y}{\partial z}=x\Big|_0^1+y\Big|_{-1}^1=1$$