MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

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POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0, f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint_{\partial K} (2x + 3)\,dxdy$. 20

3. Neka C plast cilindra koji ne ukljuèuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint_{\partial K} 2x + 3dydz$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int_C f\,ds$ 20

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int_{\widehat{\Gamma}} \frac{xdx + ydy}{\sqrt{3 + x^2 + y^2}} =$$

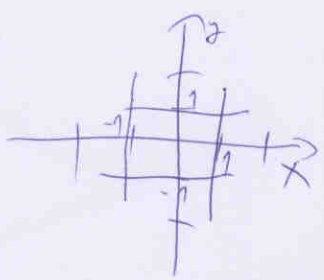
20

Ukupno:

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

2.



$$\int_{-1}^{1}\int_{-1}^{1} (2x+3)\,dx\,dy =$$

$$\int_{-1}^{1}\int_{-1}^{1} (2\cos t + 3)\,dx\,dy =$$

5)

$$\frac{x^2}{3} + \frac{y^2}{15} = 1$$

$$\int_{\Gamma} \frac{x\,dx + y\,dy}{\sqrt{3 + x^2 + y^2}} = ?$$

$\varphi \in (0, \frac{\pi}{2})$

~~$r \in (0, 4\sqrt{5})$~~

$r \in (0, \sqrt{\frac{1}{5}})$

$$\frac{r^2\cos^2\varphi}{3} + \frac{r^2\sin^2\varphi}{15} = 1$$

$$\frac{r^2}{5} = 1$$

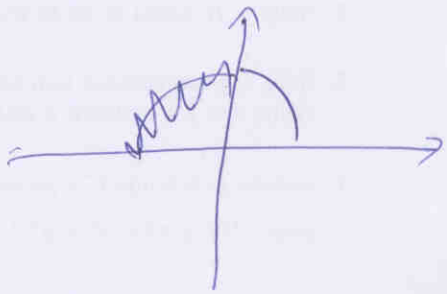
$$r^2 = \frac{1}{5} \quad | \sqrt{\ }$$

~~$r = \sqrt{5}$~~

$$r = \sqrt{\frac{1}{5}}$$

$$\frac{r^2}{5} = 1$$

$$r^2 = \frac{1}{5}$$



$$\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{\frac{1}{5}}} \frac{d\varphi\, r\, dr}{\sqrt{3 + (r\cos\varphi)^2 + (r\sin\varphi)^2}} =$$

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Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

4. $x=\cos(t)+1$, $y=\frac{t}{2}$ i $z=\sin t$

$t\in[0,4\pi]$

$$\vec{r}(t)=\begin{pmatrix}x\\y\\z\end{pmatrix}\quad \vec{r}'(t)=\begin{pmatrix}-\sin(t)\\\frac{1}{2}\\\cos\end{pmatrix}$$

$$\|\vec{r}'\|=\sqrt{(-\sin t)^2+\left(\frac{1}{2}\right)^2+(\cos)^2}$$
$$=\sqrt{(-\sin t)^2+(\cos)^2+\left(\frac{1}{2}\right)^2}$$
$$=\sqrt{1+\frac{1}{4}}$$
$$=\sqrt{\frac{5}{4}}$$ ✓

$$f(x,y,z)=(\cos(t)+1)^2+\left(\frac{t}{2}\right)^2+(\sin t)^2$$
$$=(\cos(t))^2+(\sin t)^2+1^2+\left(\frac{t}{2}\right)^2$$
$$=2+\left(\frac{t}{2}\right)^2$$
$$=2+\frac{t^2}{4}$$

$$\int_C f\,ds=\int_0^{4\pi}\sqrt{\frac{5}{4}}\cdot 2+\frac{t^2}{4}\,dt=2\sqrt{\frac{5}{4}}\int_0^{4\pi}\frac{t^2}{4}\,dt=$$

$$\left(2\sqrt{\frac{5}{4}}\cdot\frac{t^3}{3}\,dt\right)\Bigg|_0^{4\pi}=\sqrt{5}\cdot\frac{t^3}{4\cdot 3}\Bigg|_0^{4\pi}=\sqrt{5}\cdot\frac{(4\pi)^3}{12}$$

$$\sqrt{5} \cdot \frac{(4\pi)^3}{12} = (4\pi)^3 \frac{\sqrt{5}}{12}$$