MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

Grupa xxoxo

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: LUKA MARĐETKO

BROJ INDEKSA:

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$2f'''(t) + 2f''(t) = 0, \quad f'(0) = 0,\ f(0) = f''(0) = 2.$$

2. Neka je K kocka stranice duljine $a = 2$ centrirana u ishodištu. Izračunati $\iint\limits_{\partial K} (2x + 3)\, dx dy$. 20

3. Neka C plast cilindra koji ne uključuje baze (nije zatvoren), radijusa $r = 1$ koji se prostire u smjeru z–osi, visine $v = 2$ s centrom u ishodistu ($z \in [-1, 1]$). Izracunati $\iint\limits_{\partial K} 2x + 3 dy dz$. 20

4. Zadana je krivulja C s parametrizacijom $t \in [0, 4\pi]$: $x = \cos(t) + 1$, $y = \frac{t}{2}$ i $z = \sin t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izraèunati $\int\limits_C f\, ds$ ~~20~~ 15

5. Neka je $\widehat{\Gamma}$ dio pozitivno usmjerene (suprotno kazaljki na satu) elipse $\frac{x^2}{3} + \frac{y^2}{15} = 1$ u prvom kvadrantu. Izračunati

$$\int\limits_{\widehat{\Gamma}} \frac{x dx + y dy}{\sqrt{3 + x^2 + y^2}} =$$

20

Ukupno: 15

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n\, dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x\, dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x\, dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

$a = 2$

2. $\int_0^2 2x dx + 3 \int_0^2 dy$ 0

$= \int_0^2 2x dx + 3y \Big|_0^2 = \int_0^2 2x dx + 3 \cdot 2$

$= \int_0^2 2x dx + 6 = 2 \frac{x^2}{2} \Big|_0^2 + 6 = 2 \cdot \frac{2^2}{2} + 6$

$= 2 \cdot 2 + 6 = 10$

4. $f(t) = (\cos(t)+1)^2 + \left(\frac{t}{2}\right)^2 + (\sin t)^2$

$$\cancel{f(t) = \cos^2 t + 1 + \frac{t^2}{4} + \sin^2 t}$$

$$\cancel{f(t) = 1 + 1 + \frac{t^2}{4} = 2 + \frac{t^2}{4}}$$

$$\cancel{\neq \sqrt{\frac{5}{4}} \int_0^{4\pi} 2 + \frac{t^2}{4}\, dt = \sqrt{\frac{5}{4}} \int_0^{4\pi} 2 + t^2 \cdot \frac{1}{4}\, dt}$$

$$\cancel{= \sqrt{\frac{5}{4}}\ \frac{t^3}{3} \cdot \frac{1}{4} \Big|_0^{4\pi}}$$

$$\cancel{= \sqrt{\frac{5}{4}} \cdot \left(\frac{4\pi^3}{3}\right) \cdot \frac{1}{4} = 184{,}886}$$

$$\rightarrow f(t) = \cos^2 t + 2\cos t + 1 + \frac{t^2}{4} + \sin^2 t$$

$$f(t) = 1 + 1 + \frac{t^2}{4} + 2\cos t = 2 + \frac{t^2}{4} + 2\cos t$$

$$\sqrt{\frac{5}{4}} \int_0^{4\pi} 2 + \frac{t^2}{2}\,dt + 2\cos t\, dt = \sqrt{\frac{5}{4}} \cdot \left(\frac{t^3}{3} + \sin t\right)\Big|_0^{4\pi}$$

15 ?

$$= \sqrt{\frac{5}{4}} \cdot \left(\frac{1}{3} \cdot (4\pi)^3 + \sin 4\pi\right)$$

$$= \sqrt{\frac{5}{4}} \cdot \frac{(4\pi)^3}{3} + \sin 4\pi = \cancel{\sqrt{\frac{5}{4}} \cdot \frac{4\pi^3}{3}}\ \sqrt{\frac{5}{4}} \cdot \frac{4\pi^3}{3}$$

2. $a = 2$ $\iint (2x+3)\,dx\,dy$

$$\frac{\partial x}{\partial x} = 2 \qquad \frac{\partial y}{\partial y} = 0$$

$$\iint (2 - 0)\,dx\,dy$$

0

LUKA MARDETKO

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}\,f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n\,f(t)$	$(-1)^n\,F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. $2f'''(t)+2f''(t)=0$ $\quad \left\{ \begin{array}{l} f'(0)=0 \\ f(0)=f''(0)=2 \end{array} \right.$

$= 2\cdot(s^3F(s)-s^2f(0)-s\cancel{f'(0)}^{0}-f''(0))+2\cdot(s^2F(s)-sf(0)-\cancel{f'(0)}^{0})=0$

$= 2\cdot(s^3F(s)-2s^2-2)+2\cdot(s^2F(s)-2s)=0$

$= 2s^3F(s)-4s^2-4+2s^2F(s)-4=0$

$F(s)(2s^3+2s^2)=4s^2+8$

$$\frac{4s^2+8}{2s^2(s+1)}=\frac{A}{2s^2}+\frac{B}{s}+\frac{C}{s+1}$$

$4s^2+8=A\cdot(s+1)+B\cdot s\cdot 2\cdot(s+1)+C\cdot 2s^2$

$4s^2+8=As+A+\underline{2Bs^2}+2Bs+\underline{2Cs^2}$

$4=(2B+2C)$

$0=(A+2B)\rightarrow 0=8+2B$

$\boxed{8=A}$

$-8=2B\ /:2$

$\boxed{B=-4}$

$4=2\cdot(-4)+2C$

$4+8=2C$

$12=2C\ /:2$

$\boxed{C=6}$

$$= 8 \cdot \frac{1}{2s^2} - 4 \cdot \frac{1}{s} + 6 \cdot \frac{1}{s+1}$$

$$= 8 \cdot (2t) - 4 + 6e^{-t}$$

$$= 16t - 4 + 6e^{-t}$$

TREBA UVRSTITI / PROVJERITI...

4. $t \in [0, 4\pi]$ $\qquad f(x,y,z) = x^2 + y^2 + z^2$

$x = \cos(t) + 1$

$y = \frac{t}{2}$

$z = \sin t$

$$r(t) = \begin{Bmatrix} \cos(t)+1 \\ \frac{t}{2} \\ \sin t \end{Bmatrix} \quad r'(t) = \begin{Bmatrix} -\sin t \\ \frac{1}{2} \\ \cos t \end{Bmatrix}$$

$$\|r'(t)\| = \sqrt{(-\sin t)^2 + \left(\frac{1}{2}\right)^2 + (\cos t)^2}$$

$$= \sqrt{\sin^2 t + \frac{1}{4} + \cos^2 t} = \sqrt{1 + \frac{1}{4}}$$

$$= \sqrt{\frac{5}{4}}$$

$$f(t) = \underbrace{\sin^2 t + \cos^2 t}_{1} + \frac{1}{4} = 1 + \frac{1}{4} = \frac{5}{4}$$

$$\sqrt{\frac{5}{4}} \int_0^{4\pi} \frac{5}{4}\, dt = \sqrt{\frac{5}{4}} \cdot \frac{5}{4} t \Big|_0^{4\pi} = \sqrt{\frac{5}{4}} \cdot \frac{5}{4}(4\pi - 0)$$

$$= \sqrt{\frac{5}{4}} \, \frac{5}{4} \cdot \frac{4\pi}{1} = \sqrt{\frac{5}{4}} \cdot \frac{20\pi}{4} = 17{,}561$$

$$= 35{,}124$$

(3.) $r = 1$ $r \in [0, 1]$ Luka Maretić

$z \in [-1, 1]$

$\iint 2x + 3 \, dy\, dz$ $\cancel{0}$