MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. IME I PREZIME: RJEŠENJE 2 BROJ INDEKSA:	Grupa XXOOX POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Neka je S gornja polusfera radijusa $r = 1$ sa centrom u ishodištu ($z \geq 0$) i usmjerena prema gore. Preko definicije plošnog integrala izračunati $\iint_{\partial K} 3\mathrm{d}x\mathrm{d}y$. (pomoć: $\mathrm{rot}\,(3x\mathbf{j}) = 3\mathbf{k}$) 20

2. Neka je K krug radijusa $r = 1$ sa centrom u toèki $T(2,1)$. Izraèunati $\iint_K (2x+3)\,\mathrm{d}x\mathrm{d}y$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) + f'(t) = 1. \quad x(0) = 1,\ x'(0) = 1,\ x'(0) = 1.$$

$f(0)=1,\ f'(0)=1,\ f''(0)=1$

4. Neka je K krug radijusa $r = 1$ sa centrom u točki $T(0,-1)$, a ∂K kružnica orijentirana suprotno od kazaljke na satu. Izračunati $\int_{\widehat{\partial K}} (2x+3)\,\mathrm{d}y$. 20

5. Provjeri da li je $\mathbf{w}(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ potencijalno polje. Zadana je elipsa u prostoru $\widehat{\Gamma} = \{(x,y,z) : x = 1 + 2\cos t,\ y = 1 - 3\sin t,\ z = 1 - 3\sin t,\ t \in [0, 2\pi]\}$. Izraèunati $\int_{\widehat{\Gamma}} (\mathbf{w}|\mathrm{d}\mathbf{r})$. 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

3 VIDI VIŠIĆ

② $r=1$, $T(2,1)$ $\quad x = r\cos\varphi + 2$, $y = r\sin\varphi + 1$

$$\iint\limits_K 2x+3 \, dx\,dy = \int_0^{2\pi}\int_0^1 \left[2(r\cos\varphi+2)+3\right] r \, dr \, d\varphi$$

$$= \underbrace{2\int_0^{2\pi}\int_0^1 r^2 \cos\varphi \, dr \, d\varphi}_{=0} + 7\int_0^{2\pi}\int_0^1 r \, dr \, d\varphi = 7\pi$$

$S = \partial K$ .. gornja polusfera oko $T(0,0,0)$ sa $r=1$

① $\iint\limits_{\partial K} 3 \, dx\,dy = (*)$

∂K ← OVO GOVORI DA JE RIJEČ O PLOŠNOM INTEGRALU

$dx\,dy$ ← OVO GOVORI DA JE RIJEČ O 2. VRSTI INTEGRALA

$$\iint w_x \, dy\,dz + w_y \, dx\,dz + w_z \, dx\,dy$$

$$\Downarrow$$

$$w = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

PARAMETRIZACIJA:

SA DISKA NA POLUSFERU

$r(u,v) = (u, v, \sqrt{1-u^2-v^2})$

$$\frac{\partial r}{\partial u} = \begin{pmatrix} 1 \\ 0 \\ \frac{-u}{\sqrt{1-u^2-v^2}} \end{pmatrix} \qquad \frac{\partial r}{\partial v} = \begin{pmatrix} 0 \\ 1 \\ \frac{-v}{\sqrt{1-u^2-v^2}} \end{pmatrix} \qquad \vec{n} = \frac{\partial r}{\partial u} \times \frac{\partial v}{\partial u} = \begin{pmatrix} \frac{+u}{\sqrt{1-u^2-v^2}} \\ \frac{v}{\sqrt{1-u^2-v^2}} \\ 1 \end{pmatrix}$$

$$(*) = \iint\limits_D \underbrace{\begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \cdot \vec{n}}_{3} \, du\,dv = 3 \underbrace{P(D)}_{= r^2\pi} = 3\pi$$

④ K ... krug sa $r=1$, centar u $T(0,-1)$,
$\overleftarrow{\partial K}$ ... orjentacija suprotna od kazaljke na satu, cijela kružnica

$$\int_{\overleftarrow{\partial K}} (2x+3)\,dy = \left\{\begin{matrix}\text{KRIVULJNI}\\ \text{INTEGRAL}\\ \text{2. VRSTE}\end{matrix}\right\} = (*) \qquad w = \begin{pmatrix}0\\ 2x+3\end{pmatrix}$$

PARAMETRIZACIJA:

$$r(t) = \begin{pmatrix}\cos t\\ -1+\sin t\end{pmatrix},\ t\in[0,2\pi]$$

$$(*) = \int_0^{2\pi} \begin{pmatrix}0\\ 2\cos t+3\end{pmatrix}\cdot\begin{pmatrix}-\sin t\\ \cos t\end{pmatrix} dt \qquad r'(t) = \begin{pmatrix}-\sin t\\ \cos t\end{pmatrix}$$

$$= \int_0^{2\pi} (2\cos^2 t + 3\cos t)\,dt = 2\int_0^{2\pi}\cos^2 t\,dt + 3\underbrace{\int_0^{2\pi}\cos t\,dt}_{=0} = 2\int_0^{2\pi}\frac{1+\cos(2t)}{2}\,dt = 2\pi$$

ISTO MOŽE JOŠ LAKŠE PREKO GREENOVE FORMULE.

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⑤ Da li postoji $f(x,y,z)$ tako da $\left\{\begin{matrix}\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}}\\ \frac{\partial f}{\partial y} = \frac{y}{\sqrt{x^2+y^2+z^2}}\\ \frac{\partial f}{\partial z} = \frac{z}{\sqrt{x^2+y^2+z^2}}\end{matrix}\right\}$?

VIDI POGLAVLJE SEMINARA "VEKTORSKA ANALIZA" I "KRIVULJNI INTEGRAL U POTENCIJALNOM POLJU" ... $f(x,y,z) = +\sqrt{x^2+y^2+z^2}$

DALJE, $\vec{\Gamma}$ JE ELIPSA, ZATVORENA KRIVULJA, POČETAK I KRAJ SU ISTA TOČKA.

ZADANI INTEGRAL $\int_{\vec{\Gamma}} (w \mid dr) = \oint_{\vec{\Gamma}} (w \mid dr) = f(A) - f(B) = 0$

ISTO

POSEBNA OZNAKA ZA CIRKULACIJU