MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

Ime i prezime: TOMISLAV ŽRILIĆ **Broj indeksa:**

Grupa	xxoxx
Popunjava nastavnik	Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
3. Zadan je trokut s vrhovima $A(-1,0)$ $B(0,1)$ i $C(-1,-1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X u prvom kvadrantu ($x \geq 0, y \geq 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

2. $x^2 + y^2 = 5z$

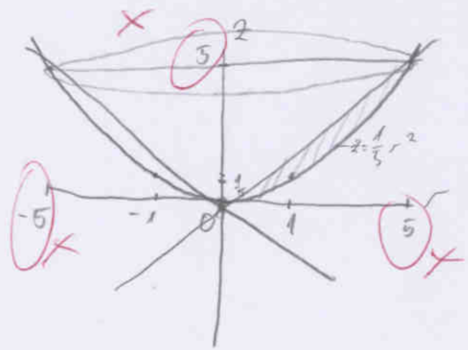
$x^2 + y^2 = r^2$

$r^2 = 5z$

$z = \frac{r^2}{5}$

$z = \frac{1}{5}r^2$

r	1	-1	0	5	-5
$z = \frac{1}{5}r^2$	$\frac{1}{5}$	$\frac{1}{5}$	0	5	5



$\varphi \in (0, 2\pi)$

$r \in (0, 5)$

$z \in (\frac{1}{5}r^2, 1)$

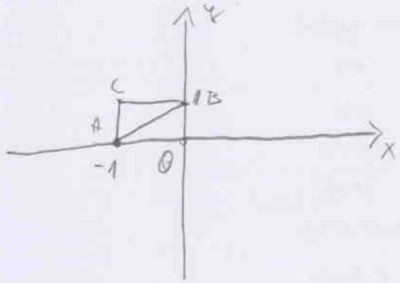
$\int_0^{2\pi} d\varphi \int_0^5 r\,dr \int_{\frac{1}{5}r^2}^1 dz$

$2\pi \int_0^5 r\left(1 - \frac{1}{5}r^2\right) dr = 2\pi \int_0^5 \left(r - \frac{1}{5}\cdot r^3\right) dr$

$2\pi \left(\frac{r^2}{2} - \frac{1}{5}\frac{r^4}{4}\right)\Big/_0^5 = 2\pi\left(\frac{5^2}{2} - \frac{1}{5}\cdot\frac{5^4}{4}\right) = 2\pi\left(\frac{25}{2} - \frac{1}{5}\cdot\frac{625}{4}\right)$

$2\pi\left(\frac{25}{2} - \frac{125}{4}\right) = 2\pi\left(\frac{50-125}{4}\right) = 2\pi\frac{75}{4} = \frac{75}{2}\pi$

3. $A(-1,0)$. $B(0,1)$, $C(-1,1)$



AB $\quad A(-1,0)_{x_1 y_1} \quad B(0,1)_{x_2 y_2}$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = \frac{1-0}{0+1}(x+1)$$

$$y = 1(x+1)$$

$$y = 1 + x$$

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TOMISLAV ZDMIĆ

1. $s^3 T(s) - s^2 t(0) - s t'(0) - t''(0) + s^2 T(s) - s t(0) - t'(0) = \frac{2}{s^2+2^2}$

$s^3 T(s) - 0 - 2s - 0 + s^2 Ts - 0 - 2 = \frac{2}{s^2+4}$

$s^3 T(s) - 2s + s^2 Ts - 2 = \frac{2}{s^2+4}$

$T(s)\,(s^3+s^2) - 2s - 2 = \frac{2}{s^2+4}$

$T(s)\,(s^3+s^2) = \frac{2}{s^2+4} + 2s \overset{+}{-} 2$

$T(s)\,(s^3+s^2) = \frac{2+2s^3+8s-2s^2-8}{s^2+4}$

$T(s)\,(s^3+s^2) = \frac{-6+2s^3+8s-2s^2}{s^2+4}$

$T(s) = \frac{-6+2s^3+8s-2s^2}{(s^3+s^2)(s^2+4)} = \frac{-6+2s^3+8s-2s^2}{s^2(s+1)(s^2+4)}$

$T(s) = \frac{-6+2s^3+8s-2s^2}{s^2(s+1)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+4} \quad / \cdot s^2(s+1)(s^2+4)$

$Ts = -6+2s^3+8s-2s^2 = As(s+1)(s^2+4) + B(s+1)(s^2+4) + Cs^2(s^2+4) + (Ds+E)s^2(s+1)$

$= As(s^3+4s+s^2+4) + B(s^3+4s+s^2+4) + C(s^4+4s) + (Ds+E)(s^3+s^2)$

$= A(s^4+4s^2+s^3+4s) + Bs^3+4Bs+Bs^2+4B + Cs^4+4Cs + Ds^4+Ds^3+Es^3+Es^2$

$= As^4+4As^2+As^3+4As+Bs^3+4Bs+Bs^2+4B+Cs^4+4Cs+Ds^4+Ds^3+Es^3+Es^2$

$= (A+C+D)s^4 + (A+B+D+E)s^3 + (4A+B+E)s^2 + (4A+4B+4C)s + 4B$

$A+C+D=0$

$A+B+D+E=2$

$A+B+E=-2$

$4A+4B+4C=8$

$4B=-6$

$B = -\frac{\cancel{6}\,3}{\cancel{4}\,2}$

$B = -\frac{3}{2}$

$4A+4B+4C=8$

$4A+\left(-\frac{3}{2}\right)+4C=8$

$4A+4C = 8+\frac{3}{2}$

$4A+4C = \frac{19}{2}$

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