MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa xxoxx
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
3. Zadan je trokut s vrhovima $A(-1,0)$ $B(0,1)$ i $C(-1,-1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X u prvom kvadrantu ($x \geq 0, y \geq 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \dfrac{\pi}{2} x. \end{cases}$ 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

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(1.) $f'''(t) + f''(t) = \sin(2t)$ | $f'(0) = 2$, $f(0) = f''(0) = 0$

$\Rightarrow s^3F(s) - s^2f(0) - 2s - f''(0)$

$\Rightarrow s^2F(s) - s\,f(0) - 2s$

$= s^3F(s) - s^2f(0) - 2s - f''(0) + s^2F(s) - sf(0) - 2s$

$s^3F(s) - 4s = \frac{2}{s^2+4}$ X

$s^3 - 4s = \frac{2}{s^2+4}$

$F(s) = \frac{2}{(s^3-4s)(s^2+4)} = \frac{2}{s(s^2-4)(s^2+4)}$ X

$F(s) = \frac{2}{s(s^2-4)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2-4} + \frac{C}{s+4} + \frac{Ds+E}{s^2+4}$

$\equiv A\cdot(s^2-4)(s^2+4) + B\cdot s\cdot(s^2+4) + (Ds+E)\cdot s\,(s^2-4) + C\cdot s(s^2-4)$

$\equiv As^4 - 16A + Bs^3 + 4Bs + Cs^2 - 4Cs + Ds^4 - 4Ds^2 + Es^3 - 4Es$

$= (A+D)\cdot s^4 + (B+C+E)\cdot s^3 + (4D)s^2 + (4B-4C-4E)\cdot s - 16A$

$-16A = 2 \quad A = -\frac{1}{8}$

$A + D = 0 \quad D = \frac{1}{8}$

$-\frac{1}{8} + D = 0$

1. $X(s) = -\frac{1}{2} \cdot \frac{1}{s} + \frac{1}{6} \cdot \frac{1}{s-2} + \frac{4}{24} \cdot \frac{1}{s+2} + \frac{1}{6} \cdot \frac{1}{s^2+2}$) x

$= -\frac{1}{2} e^{0} + \frac{1}{6} e^{-2t} + \frac{4}{24} e^{2t} + \frac{1}{6} \sin(2t)$

PROVJERA

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1.) $$\frac{2}{s(s^2-4)(s^2+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2} + \frac{Ds+E}{s^2+2}$$

$$= A\cdot(s^2-2)(s^2+2) + B\cdot s\cdot(s+2)\cdot(s^2+2) + C\cdot s\cdot (s-2)(s^2+2) + (Ds+E)\cdot s\,(s-2)(s+2)$$

$$\equiv \underline{A\cdot s^4} - 4A + \underline{Bs^4} + 2Bs^2 + 2Bs^3 + 4Bs + \underline{Cs^4} + 2Cs^2 - 2Cs^3 - 4Cs + \underline{Ds^4} - 4Ds^2 + Es^3 - 4Es$$

$$\equiv (A+B+C+D)\cdot s^4$$
$$(2B-2C+E)\cdot s^3$$
$$(2B+2C-4D)\cdot s^2$$
$$(4B-4C-4E)\cdot s$$

$$-4A = 2$$
$$A = -\frac{1}{2}$$

$$2B-2C+E=0 \quad /\cdot -2$$
$$4B-4C-4E=0$$

$$-4B+4C-2E=0$$
$$4B-4C-4E=0$$

$$-6E=0$$
$$E=0$$

$$-\frac{1}{2}+B+\frac{4}{24}+\frac{1}{6}=0$$

$$B = \frac{1}{2}-\frac{4}{24}-\frac{1}{6}$$

$$B = \frac{12-4-4}{24} = \frac{4}{24} = \frac{1}{6}$$

$$-\frac{1}{2}+B+C+D=0 \quad /\cdot -2$$
$$2B+2C-4D=0$$

$$1-2B-2C-2D=0$$
$$2B+2C-4D=0$$

$$1-6D=0$$
$$-6D=-1$$
$$D=\frac{1}{6}$$

$$-\frac{1}{2}+B+C+\frac{1}{6}=0 \quad /\cdot -4$$
$$4B-4C-0=0$$

$$2-4B-4C-\frac{2}{3}=0$$
$$4B-4C-0=0$$

$$2-8C-\frac{2}{3}=0$$

$$-8C = \frac{2}{3}-\frac{2}{1}$$

$$-8C = -\frac{4}{3}$$

$$C = \frac{4}{24}$$

(2.) $x^2+y^2=r^2$

$r^2=5z$

$r=\sqrt{5z}$

(4.) $f(x,y)=\frac{1}{\sqrt{x^2+y^2}}$ $\varphi \in [0, \frac{3\pi}{2}]$

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