MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **Pišite dvostrano!**

Ime i prezime: MAGDA MANDIĆ

Broj indeksa: 0269015993

Grupa xxoxx

Popunjava nastavnik

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t)+f''(t)=\sin(2t)$, $f'(0)=2$ i $f(0)=f''(0)=0$. — 20 / 20
2. Izračunajte površinu oplošja paraboloida $x^2+y^2=5z$, $z\le 1$. — 20
3. Zadan je trokut s vrhovima $A(-1,0)$ $B(0,1)$ i $C(-1,-1)$. Izracunati $\oint\limits_{\widehat{ABC}}(x^2-y)\,\mathrm{d}x+\sin(y^3)\,\mathrm{d}y$. — 20
4. Izračunati integral funkcije $f(x,y)=\dfrac{1}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi\in[0,\frac{3\pi}{2}]$) radijusa $r=3$ sa središtem u ishodištu. — 20
5. Odrediti integral funkcije $f(x,y)=-y$ na području X u prvom kvadrantu ($x\ge 0$, $y\ge 0$) koje je ograničeno krivuljama $X\ldots\begin{cases}x=\sin y,\\ y=\dfrac{\pi}{2}x.\end{cases}$ — 20

Ukupno: 20

Tablica integrala

$\int \mathrm{d}x = x+C$	$\int \frac{\mathrm{d}x}{\cos^2 x}=\tan x+C$	$\int \frac{\mathrm{d}x}{a^2+x^2}=\frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x}=-\cot x+C$	$\int \frac{\mathrm{d}x}{a^2-x^2}=\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert+C$
$\int \frac{\mathrm{d}x}{x}=\ln\lvert x\rvert+C$	$\int \sinh x\,\mathrm{d}x=\cosh x+C$	$\int \frac{\mathrm{d}x}{x^2-a^2}=\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert+C$
$\int a^x\,\mathrm{d}x=\frac{a^x}{\ln a}+C$	$\int \cosh x\,\mathrm{d}x=\sinh x+C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}}=\ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert+C$
$\int \sin x\,\mathrm{d}x=-\cos x+C$	$\int \tanh x\,\mathrm{d}x=\ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}}=\arcsin\frac{x}{a}+C$
$\int \cos x\,\mathrm{d}x=\sin x+C$	$\int \coth x\,\mathrm{d}x=\ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}}=\arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,\mathrm{d}x=-\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x}=\tanh x+C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{x^2\pm a^2}\pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x=\ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x}=-\coth x+C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x=\frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. $f'''(t)+f''(t)=\sin(2t) \Rightarrow F(s)$ $\quad f(0)=0 \quad f''(0)=0$

$F(t)=\mathcal{L}^{-1}F(s)$ $\quad f'(0)=2$

$s^3F(s)-s^2\cancel{f(0)}-s\cancel{f'(0)}-\cancel{f''(0)}+s^2F(s)-s\cancel{f(0)}-\cancel{f'(0)}=\frac{2}{s^2+2^2}$

$s^3F(s)-2s+s^2F(s)-2=\frac{2}{s^2+4}$

$s^3F(s)+s^2F(s)=\frac{2}{s^2+4}+2s+2$

$F(s)(s^3+s^2)=\frac{2+2s(s^2+4)+2(s^2+4)}{s^2+4}$

$F(s)(s^3+s^2)=\frac{2+2s^3+8s+2s^2+8}{s^2+4}$

$F(s)(s^2(s+1))=\frac{2s^3+2s^2+8s+10}{s^2+4} \quad \Big|:s^2(s+1)$

$F(s)=\frac{2s^3+2s^2+8s+10}{s^2(s+1)(s^2+4)}$

$2s^3+2s^2+8s+10=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+1}+\frac{Ds+E}{s^2+4}$

$2s^3+2s^2+8s+10=A(s+1)(s^2+4)+Bs(s+1)(s^2+4)+Cs^2(s^2+4)+(Ds+E)(s^2)(s+1)=$

$2s^3+2s^2+8s+10=A(s^3+4s+s^2+4)+Bs(s^3+4s+s^2+4)+Cs^4+4Cs^2+(Ds+E)(s^3+s^2)$

$2s^3+2s^2+8s+10=\underline{As^3}+\underline{4As}+\underline{As^2}+\underline{4A}+\underline{Bs^4}+\underline{4Bs^2}+\underline{Bs^3}+\underline{4Bs}+\underline{Cs^4}+\underline{4Cs^2}+\underline{Ds^4}+\underline{Ds^3}+\underline{Es^3}+\underline{Es^2}$

$0 = B + C + D \rightarrow \frac{1}{2} = C + D \rightarrow C = \frac{1}{2} - D$

$2 = A + B + D + E$

$2 = A + 4B + 4C + E$

$8 = 4A + 4B \longrightarrow 8 = 4 \cdot \frac{5}{2} + 4B$

$10 = 4A \rightarrow \boxed{A = \frac{10}{4} = \frac{5}{2}}$

$8 = 10 + 4B$

$-2 = 4B$

$\boxed{B = -\frac{1}{2}}$

$2 = \frac{5}{2} - \frac{1}{2} + D + E$

$2 = \frac{5}{2} + 4 \cdot \left(-\frac{1}{2}\right) + 4 \cdot \left(\frac{1}{2} - D\right) + E$

$2 = 2 + D + E$

$2 = \frac{1}{2} + 2 - 4D + E$

$0 = D + E \quad | \cdot (-1)$

$-\frac{1}{2} = -4D + E$

$0 = -D - \cancel{E}$

$-\frac{1}{2} = -4D + \cancel{E}$

$-\frac{1}{2} = -5D \quad | \cdot (-2)$

$1 = 10D$

$\boxed{D = \frac{1}{10}}$

$\frac{1}{2} = C + \frac{1}{10} \quad | \cdot 10$

$5 = 10C + 1$

$4 = 10C$

$\boxed{C = \frac{2}{5}}$

$2 = \frac{5}{2} - \frac{1}{2} + \frac{1}{10} + E$

$2 = 2 + \frac{1}{10}$

$\boxed{-\frac{1}{10} = E}$

$F(t) = \mathcal{L}^{-1} F(s) = \mathcal{L}^{-1}\left\{\frac{5}{2} \cdot \frac{1}{s^2} - \frac{1}{2} \cdot \frac{1}{s} + \frac{2}{5} \cdot \frac{1}{s+1} + \frac{\frac{1}{10}s - \frac{1}{10}}{s^2+4}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{5}{2} \cdot \frac{1}{s^2} - \frac{1}{2} \cdot \frac{1}{s} + \frac{2}{5} \cdot \frac{1}{s+1} + \frac{1}{10} \cdot \frac{s}{s^2+4} - \frac{1}{10} \cdot \frac{1}{s^2+4}\right\}$ $a=2$

$F(t) = \frac{5}{2} \cdot t - \frac{1}{2} \cdot 1 + \frac{2}{5} \cdot e^{-t} + \frac{1}{10} \cdot \cos(2t) - \frac{1}{10} \cdot \frac{1}{2} \cdot \sin(2t)$

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$F(t) = \frac{5}{2} \cdot t - \frac{1}{2} + \frac{2}{5} e^{-t} + \frac{1}{10} \cdot \cos(2t) - \frac{1}{20} \cdot \sin(2t)$

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2) $x^2+y^2=5z$ $\quad z \leq 1$

$(r\cos\varphi)^2+(r\sin\varphi)^2=5z$

$r^2\cos^2\varphi + r^2\sin^2\varphi = 5z$

$r^2(\cos^2\varphi+\sin^2\varphi)=5z$

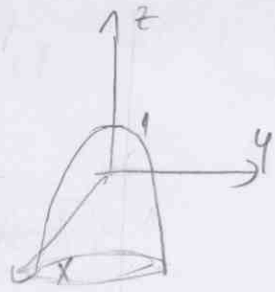
$r^2 \cdot 1 = 5z$ $\quad z=\frac{r^2}{5}$

$r=\sqrt{5z}$

$r=\sqrt{5\cdot 1}=\sqrt{5}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{5}]$



$P = \iint dx\,dy = \int d\varphi \int r\,dr$

$P = \int_0^{2\pi} d\varphi \int_0^{\sqrt5} r\,dr = \int_0^{2\pi} d\varphi \left(\frac{r^2}{2}\right)\Big|_0^{\sqrt5}$

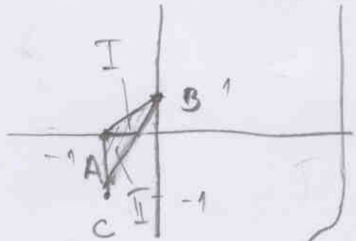
$= \int_0^{2\pi} d\varphi \left(\frac{\sqrt5^2}{5}\right) = \int_0^{2\pi} d\varphi = (\varphi)\Big|_0^{2\pi} = 2\pi$

$= 6{,}283$

3) $A(-1,0)$

$B(0,1)$

$C(-1,-1)$



$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy =$

$\left[\begin{array}{l} x^2-y \rightarrow x' = 2x-1 \\ \sin y^3 \rightarrow y' = 3\sin y^2 = 3\cos y^2 \end{array}\right.$

$\|r'(t)\| = \sqrt{(2x-1)^2+(3\cos y^2)^2}$

$= \sqrt{(4x^2-4x+1)+(9\cos^2 y^2)}$

$\|r'(t)\| =$?

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

AB: $y-0=\frac{1-0}{0+1}(x+1)$

$y = x+1 \Rightarrow \boxed{x=y-1}$

BC: $y-1=\frac{-1-1}{-1-0}(x-0)$

$y = \frac{-2}{-1}+1(x)$

$y=3x$ $\quad \boxed{x=\frac{y}{3}}$

AC: $y-0=\frac{-1-0}{-1+1}(x-0)$

$\boxed{y=0}$

$\cancel{0}$

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④ $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$ $\quad (\varphi \in [0, \frac{3\pi}{2}]) \quad r=3 \quad r \in [0,3]$

$= \frac{1}{\sqrt{r^2\cos^2\varphi + r^2\sin^2\varphi}}$

$= \frac{1}{\sqrt{r^2(\cos^2\varphi+\sin^2\varphi)}}$

$= \frac{1}{\sqrt{r^2}} = \frac{1}{\sqrt{3^2}} = \frac{1}{3}$

$f(x,y) = \int_0^{\frac{3\pi}{2}} d\varphi \int_0^3 \frac{1}{3} \cdot r\,dr = \int_0^{\frac{3\pi}{2}} d\varphi \cdot \left(\frac{1}{3} \cdot \frac{r^2}{2} dr\right)\Big|_0^3$

$= \int_0^{\frac{3\pi}{2}} d\varphi \left(\frac{1}{3} \cdot \frac{9}{2}\right) = \int_0^{\frac{3\pi}{2}} \frac{3}{2}\,d\varphi = \left(\frac{3}{2}\varphi\right)\Big|_0^{\frac{3\pi}{2}}$

$= \frac{3}{2} \cdot \frac{3\pi}{2} = \frac{9\pi}{4} = 7{,}068$ $\quad \emptyset$

⑤ $f(x,y) = -y$ $\quad (x \geq 0, y \geq 0)$

$X \ldots \begin{cases} x = \sin y \\ y = \frac{\pi}{2} x \end{cases}$ $\quad x' = \cos y$ $\quad y' = \frac{\pi}{2}$

$\emptyset$