2

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa
IME I PREZIME: ANTE GRUBIŠA **BROJ INDEKSA:** 57831	XXOXX POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. — 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \le 1$. — ~~20~~
3. Zadan je trokut s vrhovima $A(-1, 0)$ $B(0, 1)$ i $C(-1, -1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. — ~~20~~
4. Izračunati integral funkcije $f(x, y) = \dfrac{1}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. — ~~20~~ 15
5. Odrediti integral funkcije $f(x, y) = -y$ na području X u prvom kvadrantu ($x \ge 0$, $y \ge 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2} x. \end{cases}$ — 20

Ukupno: 15

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \ne -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a} F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1 - at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

2. $x^2 + y^2 = 5z \quad z \le 1$

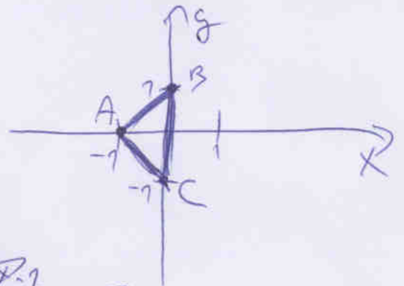
$r \in [0, \sqrt{5z}]$
$\varphi \in [0, 2\pi]$
$z \in [0, 1]$

$x = r\cos\varphi \quad r^2 = 5z \,/\sqrt{\ }$
$y = r\sin\varphi \quad r = \sqrt{5z}$

$\int_0^1 dz \int_0^{\sqrt{5z}} r\,dr \int_0^{2\pi} d\varphi =$

TRAŽI SE POVRŠINA OPLOŠJA

3. $A(-1, 0)$ $B(0, 1)$ $C(-1, -1)$



$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

AB $y - 0 = \frac{1 - 0}{0 + 1}(x + 1)$

$y = x + 1$

$y = x + 1$
$-x = -y + 1 \,/\,(-1)$
$y - 1$

AC $= y - 0 = \frac{-1 - 0}{-1 + 1}(x + 1)$

$= x + 1$

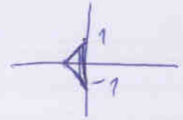
BC $= y + 1 = \frac{-1 - 1}{-1 - 0}\,x - 0$
$= 2x - 1$

~~$\int_{-1}^{1}\int_{y-1}^{y-1} (x^2 - y)\,dx\,dy + \int_{-1}^{1}\int_{y-1}^{y-1} \sin(y^3)\,dy$~~

$\int_{-1}^{1}\int_{y-1}^{y-1} (x^2 - y)\,dx\,dy + \int_{-1}^{1}\int_{y-1}^{y-1} dx\,\sin(y^3)\,dy =$

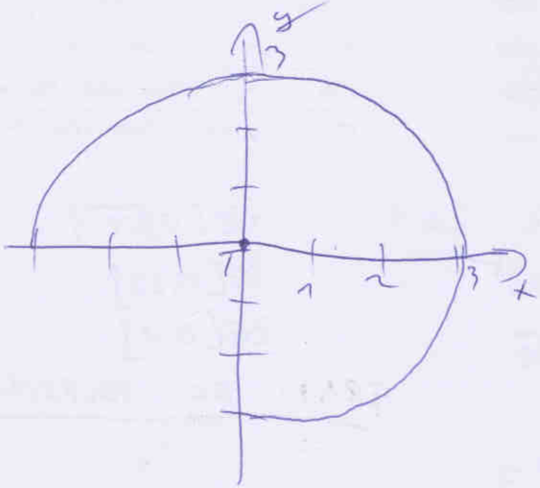
4. $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$ $\qquad r = 3$

$(r\cos\varphi)^2 \quad (r\sin\varphi)^2$



$\varphi \in \left[0, \frac{3\pi}{2}\right]$

$r \in [0, 3]$



15

$$\int_0^{\frac{3\pi}{2}} d\varphi \int_0^3 \frac{r\,dr}{\sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2}} = \int_0^{\frac{3\pi}{2}} d\varphi \int_0^3 \frac{r\,dr}{\sqrt{r^2\cos\varphi^2 + r^2\sin\varphi^2}} =$$

$$= \int_0^{\frac{3\pi}{2}} d\varphi \int_0^3 \frac{r\,dr}{r\cos\varphi + r\sin\varphi} = \int_0^{\frac{3\pi}{2}} d\varphi \int_0^3 r\,dr =$$