MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa xxoxx

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. — 20
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. — 20
3. Zadan je trokut s vrhovima $A(-1,0)$ $B(0,1)$ i $C(-1,-1)$. Izracunati $\oint_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. — 20
4. Izračunati integral funkcije $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. — 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X u prvom kvadrantu ($x \geq 0, y \geq 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \dfrac{\pi}{2}x. \end{cases}$ — 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

1.) $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$, $f(0) = f''(0) = 0$

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$$s^3F(s) - s^2\underbrace{f(0)}_{0} - s\underbrace{f'(0)}_{0} - \underbrace{f''(0)}_{0} + s^2F(s) - s\underbrace{f(0)}_{0} - \underbrace{f'(0)}_{0} = \frac{a}{s^2 - a^2}$$

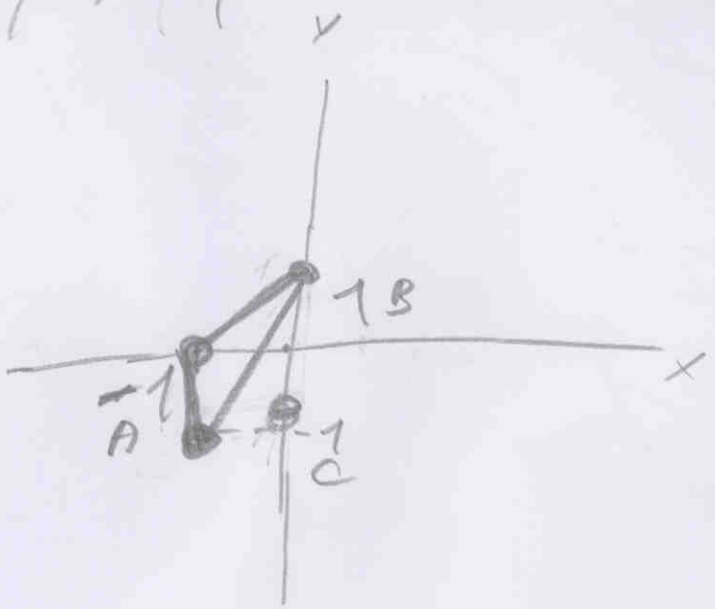
$$s^3F(s) - s^2 - s + s^2F(s) - s = \frac{a}{s^2 - a^2}$$

$$s^4 - s^2 - s + s^3 - s = \frac{a}{s^2 - a^2}$$

$$s^2 - s^4 - s = \frac{a}{s^2 - a^2}$$

3.) $\int (x^2 - y)\,dx + \sin(y^3)\,dy$

$$\int_{-1}^{0} (x^2 - y)\,dx \int_{-1}^{-1} \sin(y^3)$$



$$\int_{0}^{-1} (x^2 - y)\,dy \int_{1}^{0} dx \sin(y^3)\Big|_{-1}^{-1}$$

$$\int_{-1}^{0}(x^2-y)\,dy \int dx \sinh\frac{1}{3} - \sinh\frac{1}{3}$$

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$$\int_{-1}^{0}(x^2-y)\,dy \int_{1}^{0} x\sinh\frac{3}{3} - \sinh\frac{3}{3}$$

$$\int_{-1}^{0}(x^2-y)\,dy \quad \frac{x^2}{2}$$

$$\int_{-1}^{0}\left(x^2-y^2-\frac{x^2}{2}\right)$$

$$\frac{1^2}{2} - \frac{1^2}{2}\cdot\frac{2^2}{2}\Big|_{0}^{-1}$$

$$\frac{2^2}{2} = \frac{4}{2} = 2$$