1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. 20

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20

3. Zadan je trokut s vrhovima $A(-1,0)$ $B(0,1)$ i $C(-1,-1)$. Izracunati $\displaystyle\oint_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. 20

4. Izračunati integral funkcije $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. 20

5. Odrediti integral funkcije $f(x,y) = -y$ na području X u prvom kvadrantu ($x \geq 0$, $y \geq 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ 20

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
[illegible]	[illegible]	[illegible]

① $f'''(t)+f''(t)=\sin(2t)$

$f(0)=0,\ f'(0)=2,\ f''(0)=0$

$\xrightarrow[\text{TRANSFORMACIJA}]{\text{LAPLACEOVA}}$

$s^3F(s)-2s+s^2F(s)-2=\frac{2}{s^2+4}$

$F(s)=2\frac{s^3+s^2+4s+5}{s^2(s+1)(s^2+4)}$

$=2\left(\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{Ds+E}{s^2+4}\right)$

$s^3+s^2+4s+5=As(s+1)(s^2+4)+B(s+1)(s^2+4)+Cs^2(s^2+4)+(Ds+E)s^2(s+1)$

UVRŠTAVANJEM: $s=0 \Rightarrow 5=B\cdot1\cdot4 \Rightarrow B=\frac{5}{4}$

$s=-1 \Rightarrow 1=C\cdot1\cdot5 \Rightarrow C=\frac{1}{5}$

PREOSTAJE: $\frac{s^3+s^2+4s+5}{s^2(s+1)(s^2+4)}=\frac{A}{s}+\frac{5/4}{s^2}+\frac{1/5}{s+1}+\frac{Ds+E}{s^2+4}$

$\underbrace{\frac{s^3+s^2+4s+5}{s^2(s+1)(s^2+4)}-\frac{5/4}{s^2}-\frac{1/5}{s+1}}=\frac{A}{s}+\frac{Ds+E}{s^2+4}$

$\frac{\cancel{s}\left(-\frac{1}{5}s^3-\frac{1}{4}s^2-\frac{21}{20}s-1\right)}{s^{\cancel{2}}(s+1)(s^2+4)}=\frac{A}{s}+\frac{Ds+E}{s^2+4}$

UVRSTITI: $s=0 \Rightarrow -1=A\cdot1\cdot4 \Rightarrow A=-\frac{1}{4}$

$\underbrace{\frac{-\frac{1}{5}s^3-\frac{1}{4}s^2-\frac{21}{20}s-1}{s(s+1)(s^2+4)}+\frac{1/4}{s}}=\frac{Ds+E}{s^2+4}$

$=\frac{\cancel{s}\cdot\cancel{(s+1)}\cdot\left(\frac{1}{20}s-\frac{1}{20}\right)}{\cancel{s}\cdot\cancel{(s+1)}\cdot s^2+4} \Rightarrow D=\frac{1}{20},\ E=-\frac{1}{20}$

$F(s)=\frac{-1/2}{s}+\frac{5}{2}\cdot\frac{1}{s^2}+\frac{1}{5}\cdot\frac{1}{s+1}+\frac{1}{10}\cdot\frac{s}{s^2+4}+\frac{-1}{10}\cdot\frac{1}{s^2+4}$

INVERZNA TRANSFORMACIJA LAPLACEA

$f(t)=-\frac{1}{2}\cancel{t}+\frac{5}{2}t+\frac{1}{5}e^{-t}+\frac{1}{10}\cos(2t)-\frac{1}{20}\sin(2t)$

PROVJERA: UVRSTITI U GORNJE 4 JEDNAKOSTI

② 

$x^2+y^2=5z$

$z\le1$

PARAMETRIZACIJA SA KRUGA NA PARABOLOID

$(x,y)\xmapsto{\phi}\left(x,y,\frac{x^2+y^2}{5}\right)$

RADIJUS KRUGA $r=\sqrt{5}$

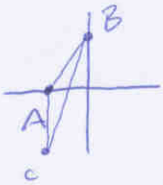
$\frac{\partial\phi}{\partial x}=\begin{bmatrix}1\\0\\\frac{2x}{5}\end{bmatrix}\quad\frac{\partial\phi}{\partial y}=\begin{bmatrix}0\\1\\\frac{2y}{5}\end{bmatrix}\quad\vec{n}=\begin{bmatrix}1\\0\\\frac{2x}{5}\end{bmatrix}\times\begin{bmatrix}0\\1\\\frac{2y}{5}\end{bmatrix}=\begin{bmatrix}-\frac{2x}{5}\\-\frac{2y}{5}\\1\end{bmatrix}$

$P=\iint_K\|\vec{n}\|=\iint_K\sqrt{\frac{4}{25}(x^2+y^2)+1}\,dx\,dy=$

$=\int_0^{2\pi}\int_0^{\sqrt5}\sqrt{\frac{4}{25}(r^2\cos^2\varphi+r^2\sin^2\varphi)+1}\ r\,dr\,d\varphi=$

$=2\pi\int_0^{\sqrt5}\sqrt{\frac{4}{25}r^2+1}\ r\,dr=\frac{2\pi}{5}\int_0^{\sqrt5}\sqrt{4r^2+25}\ r\,dr$

$=\left\{\begin{matrix}t=4r^2+25\\dt=8r\,dr\end{matrix}\right\}=\ldots=\frac{\pi}{30}\left(\sqrt{45^3}-125\right)\approx18.51$

③ 

AB: $y=x+1$

AC: $x=-1$

BC: $y=2x+1$

$\oint_{ABC}(x^2-y)dx+\sin(y^3)dy=\left\{\begin{matrix}\text{GREENOVA}\\\text{FORMULA}\end{matrix}\right\}=\underbrace{\iint_{ABC}\Big(\underbrace{\frac{\partial\sin(y^3)}{\partial x}}_{=0}-\underbrace{\frac{\partial(x^2-y)}{\partial y}}_{=-1}\Big)dx\,dy}_{=P(\triangle ABC)}=\frac{1}{2}$

④ $\Omega=$ 

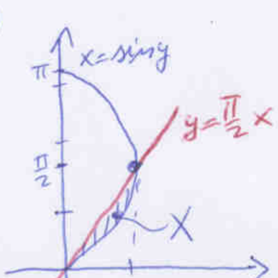
$r=3$

$\varphi\in[0,\frac{3\pi}{2}]$

$\iint_\Omega\frac{1}{\sqrt{x^2+y^2}}=\int_0^{\frac{3\pi}{2}}\int_0^3\frac{1}{r}\cdot r\,dr\,d\varphi=\frac{3\pi}{2}\cdot3=\frac{9\pi}{2}$

$x=r\cos\varphi$

$y=r\sin\varphi$

⑤ 

TRAŽI SE: $\iint_X-y\,dx\,dy=\int_0^1\int_{\arcsin x}^{\frac{\pi}{2}x}-y\,dy\,dx=\ldots$ TEŽE

$=\int_0^{\frac{\pi}{2}}\int_{\frac{2}{\pi}y}^{\sin y}-y\,dx\,dy=-\int_0^{\frac{\pi}{2}}y\sin y-\frac{2}{\pi}y^2\,dy=\left[\cancel{y\cos y}-\cancel{\sin y}-y+\frac{2}{\pi}\frac{y^3}{3}\right]_{y=0}^{y=\frac{\pi}{2}}$

$\approx-1+\frac{\pi^2}{12}\approx-0.18$

PARCIJALNA INTEGRACIJA

$\int y\sin y=\left\{\begin{matrix}u=y & du=dy\\dv=\sin y & v=-\cos y\end{matrix}\right\}$

$=-y\cos y+\sin y$