MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. PIŠITE DVOSTRANO!		Grupa XXXXX
IME I PREZIME: Mateja Mitković	BROJ INDEKSA: 0269037541	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 2$ i $f(0) = f''(0) = 0$. — 20 — 0
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. — 20
3. Zadan je trokut s vrhovima $A(-1, 0)$ $B(0, 1)$ i $C(-1, -1)$. Izracunati $\oint_{\hat{ABC}} (x^2 - y)\,dx + \sin(y^3)\,dy$. — 20
4. Izračunati integral funkcije $f(x, y) = \frac{1}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 3$ sa središtem u ishodištu. — 20
5. Odrediti integral funkcije $f(x, y) = -y$ na području X u prvom kvadrantu ($x \geq 0, y \geq 0$) koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ — 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1 - at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

1. $f'''(t) = s^3F(s) - s^2\underbrace{f(0)}_{0} - s\underbrace{f'(0)}_{2} - \underbrace{f''(0)}_{0} = s^3F(s) - 2s$

$f''(t) = s^2F(s) - s\underbrace{f(0)}_{0} - \underbrace{f'(0)}_{2} = s^2F(s) - 2$

$s^3F(s) - 2s + s^2F(s) - 2 = \frac{a}{s^2+a^2} \cdot \left(2 \cdot \frac{1}{s^2}\right)$ X

ZABORAVILI POSTAVITI $a=2$

$s^3F(s) - 2s + s^2F(s) - 2 = \frac{2}{s^2 \cdot (s^2+a^2)}$

$(s^3 + s^2)F(s) = \frac{2}{s^2 \cdot (s^2+a^2)} + 2s + 2$

$(s^3 + s^2)F(s) = \frac{2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2}{s^2 \cdot (s^2+a^2)} / \cdot (s^3 + s^2)$

$F(s) = \frac{2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2}{s^2(s^2+a^2)} \cdot \frac{1}{(s^3+s^2)}$

$F(s) = \frac{2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2}{s^2(s^2+a^2) \cdot (s^3+s^2)} / \cdot s^2(s^2+a^2) \cdot (s^3+s^2)$

$2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2 = \frac{As+B}{s^2} + \frac{Bs+C}{s^2+a^2} + \frac{Cs+D}{(s^3+s^2)}$

$2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2 = As + B\,(s^2+a^2) \cdot (s^3+s^2) + Bs + C\,(s^2) \cdot (s^3+s^2)$
$+ Cs + D\,(s^2)(s^2+a^2)$

$2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2 = As + B\,(s^5 + s^4 + a^2s^3 + a^2s^2) + Bs + C\,(s^5 + s^4)$
$+ Cs + D\,(s^4 + s^2a^2)$

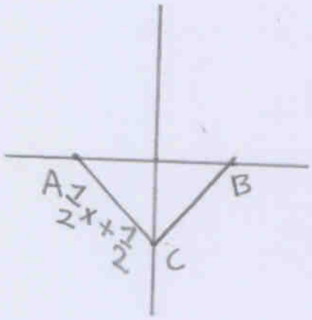
$2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2 = As^6 + As^5 + As^4a^2 + As^3a^2 + Bs^5 + Bs^4 + Ba^2s^3$
$+ Ba^2s^2 + Bs^6 + Bs^5 + Cs^5 + Cs^4 + Cs^5 + Cs^3a^2 + Ds^4 + Ds^2a^2$

$2s^5 + 2s^4 + 2s^3a^2 + 2a^2s^2 = (As^6 + As^5 + As^4a^2 + As^3a^2) + (2Bs^5 + Bs^4$
$+ Ba^2s^3 + Ba^2s^2 + Bs^6) + (2Cs^5 + Cs^4 + Cs^3a^2) + Ds^4 + Ds^2a^2$

Mateja Mitović

③

$A(-1,0)\ B(0,1)\ C(-1,-1)$



$A(-1,0)\ C(-1,-1)$

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_0)$

$y-0=\frac{-1-0}{-1-1}(x+1)$

$y-0=\frac{1}{2}(x+1)$

$y=\frac{1}{2}x+\frac{1}{2}\ |AC|$

$B(0,1)\ C(-1,-1)$

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1=\frac{-1-1}{-1-0}(x-0)$

$y-1=2(x-0)$

$y=2x+1\ |BC|$

$\cancel{\emptyset}$

JEDNADŽBE PRAVACA ZASEBNO SE NE BODUJU

②

$x^2+y^2=5z\quad z\leqslant 1$

$x^2+y^2=r^2\ (z=1)$

$x^2+y^2=5\cdot 1$

$x^2+y^2=5$

$r^2=5\quad r=\sqrt{5}$

$5z=r^2 /:5$

$z=\frac{r^2}{5}$

$\varphi\,(0,2\pi)$

$r\,(0,\sqrt{5})$

$z\,(1,\frac{r^2}{5})$

$$\int_0^{2\pi}\int_0^{\sqrt{5}}\int_1^{\frac{r^2}{5}} r\,dr\,d\varphi\,dz$$

$$\int_0^{2\pi}\int_0^{\sqrt{5}} r\,dr\,d\varphi\,\Big|_1^{\frac{r^2}{5}}=\int_0^{2\pi}\int_0^{\sqrt{5}} r\cdot\left(\frac{r^2}{5}-1\right)dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{\sqrt{5}}\left(\frac{r^3}{5}-r\right)dr\,d\varphi=\int_0^{2\pi}d\varphi\cdot\left(\frac{r^4}{20}-\frac{r^2}{2}\right)\Big|_0^{\sqrt{5}}$$

$$\int_0^{2\pi}d\varphi\cdot\left(\frac{(\sqrt{5})^4}{20}-\frac{(\sqrt{5})^2}{20}\right)=\int_0^{2\pi}d\varphi\cdot 1=1\int_0^{2\pi}d\varphi=1\cdot\varphi\Big|_0^{2\pi}$$

$$=1\cdot 2\pi=2\pi$$

$\cancel{\emptyset}$

TRAŽI SE POVRŠINA OPLOŠJA.

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IME I PREZIME: VICE VIŠIĆ **BROJ INDEKSA:** 57102

Grupa XXOXX POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 15

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$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
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t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

$f'''(t) + f''(t) = \sin 2t \qquad f'(0) = 2 \quad f(0) = f''(0) = 0$

$s^3F(s) - s^2 f(0) - sf'(0) - f''(0) + s^2F(s) - sf(0) - f'(0) = \frac{2}{s^2-2^2}$ ✗

$s^3F(s) - 2s + s^2F(s) - 2 = \frac{2}{s^2-4}$

$s^3F(s) + s^2F(s) = \frac{2}{s^2-4} + 2s + 2$

$F(s)(s^3+s^2) = \frac{2 + 2s(s^2-4) + 2(s^2-4)}{s^2-4} = \frac{2 + 2s^3 - 8s + 2s^2 - 8}{s^2-4}$

$F(s)(s^3+s^2) = \frac{2s^3+2s^2-8s-6}{s^2-4}$

$F(s) = \frac{2s^3+2s^2-8s-6}{s^2-4} \cdot \frac{1}{s^3+s^2} = \frac{2s^3+2s^2-8s-6}{(s^2-4)(s^3+s^2)}$

$F(s) = \frac{2s^3+2s^2-8s-6}{(s^2-4)(s^3+s^2)} = \frac{2s^3+2s^2-8s-6}{s^2(s+1)(s^2-4)}$

$2s^3+2s^2-8s-6 = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1} + \frac{Ds+E}{s^2-4}$

$2s^3+2s^2-8s-6 = A(s+1)(s^2-4) + Bs(s+1)(s^2-4) + Cs^2(s^2-4) + (Ds+E)(s^2)(s+1)$

$2s^3+2s^2-8s-6 = A(s^3-4s+s^2-4) + Bs(s^3-4s+s^2-4) + Cs^4-4Cs^2 + DsE(s^3+s^2)$

$2s^3+2s^2-8s-6 = As^3-4As+As^2-4A+Bs^4-4Bs^2+Bs^3-4Bs+Cs^4-4Cs^2+Ds^4+Ds^3+Es^3+Es^2$

$2s^3+2s^2-8s-6 = (B+C+D)s^4 + (A+B+D+E)s^3 + (A-4B-4C+E)s^2 + (-4A-4B)s - 4A$

$B+C+D=0$

$A+B+D+E=2$

$A-4B-4C+E=2$

$-4A-4B=-8$

$-4A=-6 \;/:(-4)$

$\boxed{A=\frac{6}{4}}$

$-4A-4B=-8 \;/:(-4)$

$A+B=4$

$\frac{6}{4}+B=4$

$B=\frac{16}{4}-\frac{6}{4}$

$\boxed{B=\frac{10}{4}}$

$A+B+D+E=2$

$4+D+E=2$

$D=-2-E$

$A-4B-4C+E=2$

$\frac{6}{4}-\frac{40}{4}-4C+E=2$

$-\frac{34}{4}-4C+E=2$

$-4C+E=\frac{8}{4}+\frac{34}{4}$

$-4C+E=\frac{42}{4}$

$B+C+D=0$

$\frac{10}{4}+C-2-E=0$

$\frac{10}{4}-\frac{8}{4}+C-E=0$

$\frac{2}{4}+C-E=0$

$\frac{1}{2}+C=E$

$D = -2 - E$

$E = \frac{1}{2} + C$

$E = \frac{42}{4} + 4C$

$A = \frac{6}{4}$

$B = \frac{10}{4}$

$E = \frac{1}{2} + C \quad | \cdot (-4)$

$E = \frac{42}{4} + 4C$

$-4E = -2 - 4C$

$E = \frac{42}{4} + 4C$

$-3E = \frac{34}{4} \quad | \cdot (-\frac{1}{3})$

$E = -\frac{34}{12} = -\frac{17}{6}$

$E = \boxed{-\frac{17}{6}}$

VICE VISE

$D = -2 - E$

$D = -\frac{12}{6} - \frac{17}{6}$

$\boxed{D = -\frac{29}{6}}$

$B + C + D = 0$

$\frac{10}{4} + C - \frac{29}{6} = 0$

$2{,}5 - 4{,}8 = C$

$\boxed{C = 2{,}33 = \frac{23}{10}}$

$f(n) = \frac{6}{4} \cdot t + \frac{10}{4} + 1 + 2{,}33 \cdot e^{-t} - \frac{29}{6} \cos - \frac{17}{6} \sin(2t)$

$f(n) = \frac{6}{4}t + \frac{10}{4} + 2{,}33 e^{t} - \frac{29}{6} \cos - \frac{17}{6} \sin(2t)$ X

2. $x^2 + y^2 = 5z \quad z \leq 1$

$r^2 = 5z$

$r = \sqrt{5z}$

$\varphi \in [0, 2\pi]$

$r \in [0, \sqrt{5z}]$

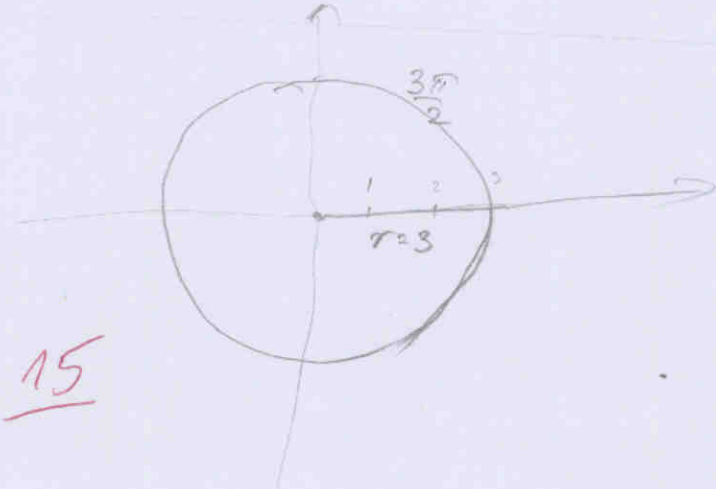
$z \in [1, 5]$

$P = \int_0^{2\pi} d\varphi \int_1^{5} dz \int_0^{\sqrt{5z}} r\,dr$ X

4. $\varphi \in [0, \frac{3\pi}{2}] \quad r = 3$

$r \in [0, 3]$

$f(x,y) = \int_0^{\frac{3\pi}{2}} d\varphi \int_0^{3} r\,dr \frac{1}{\sqrt{(r\cos\varphi)^2 + (r\sin\varphi)^2}}$ 15



5. $(x \geq 0, \ y \geq 0)$ X $\begin{cases} x = \sin y \\ y = \frac{\pi}{2} x \end{cases}$ X

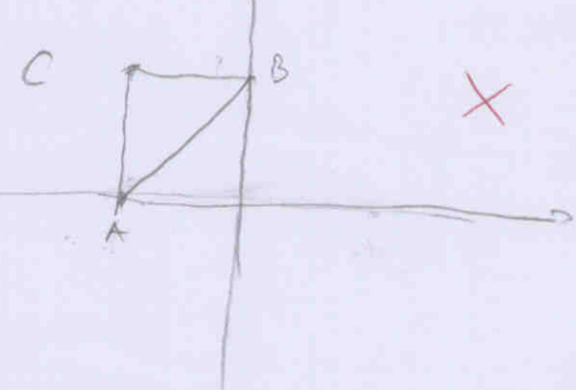
3. $A(-1,0)$ $B(0,1)$ $(-1,1)$ X

$\overline{AB}$ $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{1 - 0}{0 + 1}(x - 0)$

$y = x$

$C(-1,-1)$



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