1. Izračunati dvostruki integral $\iint\limits_S x + e^y \mathrm{d}x\mathrm{d}y$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 1$ i plohama $z = 1 - y^2$ i $z = x^2 - 1$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 0,\ x'(0) = 4.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \leq 1,\ -1 \leq z \leq 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial C}} x \,\mathrm{d}y\mathrm{d}z$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} (3x^2 + y)\,\mathrm{d}x + (3y^2 + x)\,\mathrm{d}y$ 20

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x \,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x \,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x \,\mathrm{d}x = -\cos x + C$	$\int \tanh x \,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x \,\mathrm{d}x = \sin x + C$	$\int \coth x \,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1 - at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

③ $x'''(t) + x'(t) = 0$

$x(0)=0,\ x'(0)=4,\ x''(0)=0$ } $\xrightarrow{\text{PRIJELAZ U LAPLACEOVE VARIJABLE}}$ $s^3X(s) - 4s + sX(s) = 0$

$X(s) = \dfrac{4}{s^2+1}$

INVERZNA LAPLACE TRANSFORMACIJA

LAKO SE PROVJERI DA ZADOVOLJAVA SVE 4 GORNJE JEDNADŽBE:

$x(t) = 4\sin(t)$

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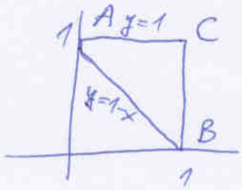
① S je trokut ABC, uz $A(0,1)$, $B(1,0)$, $C(1,1)$

IZRAČUNATI: $\iint_S x + e^y \, dx\,dy = (*)$



$$(*) = \int_0^1 \int_{1-x}^1 x + e^y \, dy\,dx = \int_0^1 \Big[xy + e^y\Big]_{y=1-x}^{y=1} dx = \int_0^1 \underbrace{x + e - x(1-x) - e^{1-x}}_{=\cancel{x} + e - \cancel{x} + x^2 - e\cdot e^{-x}} dx$$

$$= x^2 + e - e\cdot e^{-x}$$

$$= \left[\frac{x^3}{3} + ex + e\cdot e^{-x}\right]_0^1 = \frac{1}{3} + e + 1 - e = \frac{4}{3}$$

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⑤ $$\int_{(1,0)}^{(e,\pi)} (3x^2+y)\,dx + (3y^2+x)\,dy = ?$$

OVO $\int_{\text{TOČKA}}^{\text{TOČKA}}$ UPUĆUJE NA KRIVULJNI INTEG. U POTENCIJALNOM POLJU KOJI JE NEOVISAN O PUTU

POTENCIJALNO POLJE F je takvo da $\left.\begin{array}{l}\partial_x F = 3x^2 + y \\ \partial_y F = 3y^2 + x\end{array}\right\} \Rightarrow F = x^3 + xy + y^3$

REZULTAT $= F(e,\pi) - F(1,0) = e^3 + e\pi + \pi^3 - 1$

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④ C cilindar $(x+2)^2 + (y-3)^2 \le 1$, $-1 \le z \le 1$

$$\oiint_{\partial C} x\,dy\,dz = \oiint_{\partial C} \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} dS = \iiint_C \underbrace{\operatorname{div}\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}}_{=1} = \iiint_C 1 = V(C) = 2\pi$$

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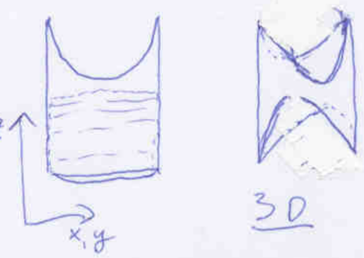
② VOLUMEN TIJELA: $\left\{\begin{array}{ll} x^2+y^2=1 & \text{VALJČASTA PLOHA} \\ z = 1-y^2 & \text{PARABOLIČNA PLOHA} \\ z = x^2-1 & \text{PARABOLIČNA PLOHA}\end{array}\right.$



$$V = \iiint 1 \, dx\,dy\,dz = \int_{r^2\cos^2\varphi - 1}^{1 - r^2\sin^2\varphi} \int_0^1 \int_0^{2\pi} r \, d\varphi \, dr \, dz = \ldots$$