

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PISITE DVOSTRANO!**

Ime i prezime: **MARIN MARKAS** Broj indeksa: **57664**

Grupa xxxxx  
POPUNJANA  
NASTAVNIK  
Broj ↓  
bodova

$$3. \quad X'''(t) + X'(t) = 0$$

$$X(0) = 0$$

$$X'(0) = 0$$

$$X''(0) = 4$$

1. Izračunati dvostruki integral  $\iint_S x + e^x dy dz$ , gdje je  $S$  trokut s vrhovima  $A(0, 1), B(1, 0), C(1, 1)$ . 20

2. Izračunati volumen tijela omeđenog vanjskom  $x^2 + y^2 = 1$  i plohanom  $z = 1 - y^2$  i  $z = x^2 - 1$ . 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:  
 $x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 0, \quad x'(0) = 4.$  20

4. Neka je  $C$  cilindar zadat sa  $C = \{(x, y, z) : (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$ . Izračunati plošni integral  $\iint_C x \, dy dz$  20

5. Izračunati  $\int_{(1,0)}^{(0, \pi)} (3x^2 + y) \, dx + (3y^2 + x) \, dy$  20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln  \cosh x $	$\frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln  \sinh x $	$\frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x \, dx = -\ln  \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[ x \sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln  \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[ x \sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$e^{-at}$	$\frac{1}{s+a}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$t^n$	$\frac{n!}{s^{n+1}}$	$e^{-at} f(t)$	$F(s+a)$
$\frac{1}{\sqrt{t}}$	$\frac{1}{\sqrt{s}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$e^{-at}$	$\frac{1}{s+a}$	$f(t)$	$F(s)$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{1}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

Ukupno:

Marvin Maras

$$3. x'''(t) + x'(t) = 0$$

$$x(0) = 0$$

$$x''(0) = 0$$

$$x'(0) = 4$$

$$\Delta^3 X(\Delta) - \cancel{\Delta^2 X(0)} - \Delta X'(0) - \cancel{x''(0)} + \Delta X(\Delta) - \cancel{x(0)} = 0$$

$$\Delta^3 X(\Delta) - 4\Delta + \Delta X(\Delta) = 0$$

$$\Delta^3 X(\Delta) + \Delta X(\Delta) = 4\Delta$$

$$X(\Delta) (\Delta^3 + \Delta) = 4\Delta$$

$$X(\Delta) = \frac{4\Delta}{\Delta^3 + \Delta}$$

$$X(\Delta) = \frac{4\Delta}{\Delta(\Delta^2 + 1)}$$

$$\frac{4\Delta}{\Delta(\Delta^2 + 1)} = \frac{A}{\Delta} + \frac{B\Delta + C}{\Delta^2 + 1}$$

$$4\Delta = A(\Delta^2 + 1) + B\Delta^2 + C\Delta$$
$$= A\Delta^2 + A + B\Delta^2 + C\Delta$$

$$0 = A + B \rightarrow \boxed{B = 0}$$

$$\boxed{4 = C}$$

$$\boxed{0 = A}$$

$$-C = A$$

$$X(\Delta) = \frac{0}{\Delta} + \frac{4}{\Delta^2 + 1} \quad \checkmark$$

$$X(\Delta) = \frac{4}{\Delta^2 + 1}$$

$$X(t) = \frac{4}{t^2 + 1}$$

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- Izračunati dvostruki integral  $\iint_S x + e^{tdz}$ , gdje je  $S$  trokut s vrhovima  $A(0,1), B(1,0), C(1,1)$ . 20
- Izračunati volumen tijela omeđenog valjkom  $x^2 + y^2 = 1$  i plohanom  $z = 1 - y^2$  i  $z = x^2 - 1$ . 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:  
 $x'''(t) + x'(t) = 0, \quad x(0) = x'(0) = 0, \quad x''(0) = 4$ . 20
- Neka je  $C$  cilindar zadan sa  $C = \{(x, y, z) : (x, y)^2 \leq 1, -1 \leq z \leq 1\}$ . Izračunati plosni integral  $\iint_C x y dz$ . 20

5. Izračunati  $\int_{(1,0)}^{(e,\pi)} (3x^2 + y) dx + (3y^2 + x) dy$

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2}  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left( x + \sqrt{x^2 \pm a^2} \right) \right] + C$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t <sup>n</sup>	$\frac{n!}{s^{n+1}}$	f(a t)	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\sqrt[n]{t}$	$\frac{1}{s^{n+1}}$	f''(t)	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{d}{dt} f(t)$	$sF(s) - f(0)$
t e^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
(1-at)e^{-at}	$\frac{1}{(s+a)^2}$	f'(t)	sF(s) - f(0)
\sin(at)	$\frac{a}{s^2 + a^2}$	f''(t)	s^2 F(s) - s f(0) - f'(0)
\cos(at)	$\frac{s}{s^2 + a^2}$	f'''(t)	s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)

20  
Ukupno:

**MATEMATIKA 3:** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnosti odgojivosti studenata. **PISITE DVOSTRANO!**

IME I PREZIME: Igor Brajica BROJ INDEKSA: 52809-2005

Grupa: XXXXX  
 POPUNJAVNA: XXXXX  
 NASTAVNIK: XXXXX  
 Broj ↓  
 bodova

1. Izračunati dvostruki integral  $\iint_S x + e^y dz dy$ , gdje je  $S$  trokut s vrhovima  $A(0, 1), B(1, 0), C(1, 1)$ . 20
2. Izračunati volumen tijela omeđenog valjkom  $x^2 + y^2 = 1$  i plohanu  $z = 1 - y^2$  i  $z = x^2 - 1$ . 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:  
 $x'''(t) + x'(t) = 0, x(0) = x''(0) = 0, x'(0) = 4$ . 20
4. Neka je  $C$  cilindar zadan sa  $C = \{(x, y, z) : (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$ . Izračunati plosni integral  $\iint_C x dy dz$ . 20

5. Izračunati  $\int_{(1,0)}^{(e, e)} (3x^2 + y) dx + (3y^2 + x) dy$  (1,0)

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int dx = \frac{1}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln  x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
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$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln  \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
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$\frac{1}{\sqrt{t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\mathcal{L}\{f\}$	$\int_0^\infty f(t) dt$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^x f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{1}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - s f'(0) - f''(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f'(0) - s f''(0) - f'''(0)$

Ukupno: 20

3)  $x'''(t) + x'(t) = 0$   
 $x(0) = 0$   
 $x'(0) = 0$   
 $x''(0) = 4$

$s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) = 0$   
 $s^3 X(s) - 0 - 0 - 4 = 0$   
 $s^3 X(s) = 4$

$s^3 X(s) - 4s - s X(s) = 0$   
 $s^3 X(s) - s X(s) = 4s$

$X(s) (s^3 - s) = 4s$   
 $X(s) (s^2 - 1) = 4$

$X(s) = \frac{4s}{s^2 - 1} = \frac{4s}{(s-1)(s+1)}$

$X(s) = \frac{4s}{s^2 - 1} = \frac{As + B}{s-1} + \frac{Cs + D}{s+1}$

$4s = A(s-1) + B(s+1) + C(s-1) + D(s+1)$

$4s = As - A + Bs + B + Cs - C + Ds + D$

$0 = A + B$   
 $0 = 0 + B$   
 $B = 0$

$4 = C$   
 $C = 4$

$0 = -A$   
 $A = 0$

$4 = \frac{4s}{s^2 - 1} = -4 \cos t$

1. Izračunati dvostrani integral  $\iint_S x + e^y dy$ , gdje je  $S$  trokut s vrhovima  $A(0,1), B(1,0), C(1,1)$ . 20

2. Izračunati volumen tijela omeđenog valjkom  $x^2 + y^2 = 1$  i plohanu  $z = 1 - y^2$  i  $z = x^2 - 1$ . 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:  
 $x'''(t) + x'(t) = 0, x(0) = x''(0) = 0, x'(0) = 4$ . 20

4. Neka je  $C$  cilindar zadan sa  $C = \{(x, y, z) : (x + 2)^2 + (y - 3)^2 \leq 1, -1 \leq z \leq 1\}$ . Izračunati plošni integral  $\iint_C x dy dz$ . 20

5. Izračunati  $\int_{(1,0)}^{(0,\pi)} (3x^2 + y) dx + (3y^2 + x) dy$ . 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left  x + \sqrt{x^2 \pm a^2} \right  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 + a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 + a^2} + a^2 \ln \left( x + \sqrt{x^2 + a^2} \right) \right] + C$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 + a^2}$
$e^{at}$	$\frac{1}{s-a}$	$\cosh(at)$	$\frac{s}{s^2 + a^2}$
$t$	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{t}$	$-\ln s$	$f^n f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{1}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$



**MATEMATIKA 3.** Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnosti odgovornosti studenata. **PISITE DVOSTRANO!**

IME I PREZIME: **MATE BALDAK**

BROJ INDEKSA: **57115**

Grupa xxxxx  
POPUNJAVNA NASTAVNIK Broj 1 bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1, \quad f'(0) = f''(0) = 0.$$

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2. Izračunati  $\int_{\partial K} \mathbf{F} \cdot d\mathbf{S}$  gdje je  $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$  i  $\partial K$  rub kugle  $K$  radijusa 2 s centrom u točki  $T(-1, 2, 0)$ , a koji je orijentiran vanjskom normalom.

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3. Izračunati volumen tijela omeđenog valjkom  $x^2 + z^2 = 1$  i ravninama  $z = y + 2$  i  $y = x^2$ .

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4. Zadata je kriviza uzvojnica  $(C)$  s parametrijom  $t \in [0, 2\pi]$ :  $x = \cos 2t$ ,  $y = \sin 2t$  i  $z = t$ . Zadatao je skalarno polje:  $f(x, y, z) = x^2 + y^2 + z^2$ . Izračunati  $\int_C f \, ds$

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5. Izračunati  $\int_{\triangle ABC} \mathbf{r} \, dx + y \, dy$  gdje je  $\triangle ABC$  kvadrata koja leži bridovima trokuta s vrhovima  $A(0, 0, 0)$ ,  $B(1, 0, 0)$ ,  $C(0, 1, 0)$  usmjerenom redom od vrha  $A$  preko  $B$  i  $C$  do ponovo vrha  $A$ . Koristiti Stokesovu formulu.

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Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2}  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm a^2 \ln x + \sqrt{x^2 \pm a^2} $
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x \sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Ukupno:

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$e^{-at}$	$\frac{1}{s+a}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$t^n$	$\frac{n!}{s^{n+1}}$	$e^{-at} f(t)$	$F(s+a)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{s}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$e^{-at}$	$\frac{1}{s+a}$	$f'(t)$	$-sF(s) - f(0)$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{1}{(s+a)^2}$	$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$		

MATEMATIKA 3: Ispit se održava sličnimo objavljenim pravilima. Na snazi je Pravilnik o strogosti odgovornosti studenata. **PISME DVOSTRANO!**  
 IME I PREZIME: **GREGOR HAHARIC** BROJ INDEKSA: **57650**  
 Grupa:   
 skok:   
 POPUNJIVA:   
 NASTAVNIK:   
 Broj 1:   
 bodova:

- Izračunati dvostruki integral  $\iint_S x + e^y dz dy$ , gdje je  $S$  trokut s vrhovima  $A(0, 1)$ ,  $B(1, 0)$ ,  $C(1, 1)$ . 20
- Izračunati volumen tijela omeđenog valjkom  $x^2 + y^2 = 1$  i plohama  $z = 1 - y^2$  i  $z = x^2 - 1$ . 20
- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednačinu:  $x''(t) + x'(t) = 0$ ,  $x(0) = x'(0) = 0$ ,  $x'(0) = 4$ . 20

4. Neka je  $C$  cilindar zadan sa  $C = \{(x, y, z) : (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$ . Izračunati plošni integral  $\iint_{\partial C} x dy dz$ . 20

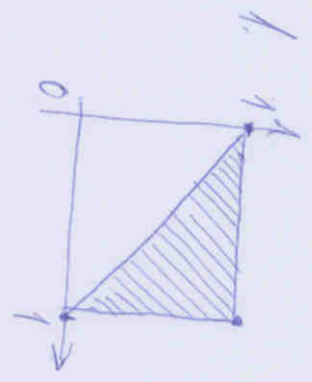
5. Izračunati  $\int_{(1,0)}^{(e,\pi)} (3x^2 + y) dx + (3y^2 + x) dy$ . 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + C$
$\int \frac{dx}{x} = \ln x  + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left  \frac{x-a}{x+a} \right  + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2}  + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left( 1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 + a^2} dx = \frac{1}{2} \left[ x\sqrt{x^2 + a^2} + a^2 \ln(x + \sqrt{x^2 + a^2}) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[ x\sqrt{a^2 - x^2} + a^2 \arcsin \left( \frac{x}{a} \right) \right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$	$f(t)$	$F(s) = \mathcal{L}\{f\}(s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{e^{as} - e^{-as}}{2a}$
$c$	$\frac{c}{s}$	$\cosh(at)$	$\frac{e^{as} + e^{-as}}{2a}$
$t$	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
$t^n$	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$e^t f(t)$	$(-1)^n F^{(n)}(s)$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{d}{dt} f(t)$	$sF(s) - f(0)$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{1}{(s+a)^2}$	$f(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2 F(s) - sf'(0) - f''(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f'(0) - sf''(0) - f'''(0)$



20  
 Ukupno