MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: NINO MIKULANDRA BROJ INDEKSA: 57645

Grupa xxoox — POPUNJAVA NASTAVNIK — Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S x + e^y \mathrm{d}x\mathrm{d}y$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. — 20
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=1$ i plohama $z=1-y^2$ i $z=x^2-1$. — 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20
$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=0,\ x'(0)=4.$$
4. Neka je C cilindar zadan sa $C=\{(x,y,z):(x+2)^2+(y-3)^2\le 1,\ -1\le z\le 1\}$. Izračunati plošni integral — 20
$$\iint\limits_{\partial C} x\,\mathrm{d}y\mathrm{d}z$$
5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} (3x^2+y)\,\mathrm{d}x + (3y^2+x)\,\mathrm{d}y$ — 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

3.) $x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 0,\ x'(0) = 4$

$$\mathcal{L}[x'''(t)] + \mathcal{L}[x(t)] = \mathcal{L}[0]$$

$$\mathcal{L}[x'''(t)] = s^2F(s) - s^2 f(0) - s f'(0) - f''(0)$$

$$\mathcal{L}[x'(t)] = sF(s) - f(0)$$

$$\mathcal{L}(t) = s^2F(s) - 0 - 4 - 0 + sF(s) - 4$$

$$\mathcal{L}(t) = s^2F(s) - 4 - 4$$

$$\mathcal{L}(t) = -\frac{8}{s^2}$$

$$s^2x(s) - s^2 - 0 - 4 - 0\ (s x(s) - 4)$$

$$s^2x(s) - s^2 - 4s x(s) = -\frac{8}{s^2}$$

$$x(s)(s^2 - 4s) = -\frac{8}{s^2} - \frac{s^2}{1} - \frac{4}{1}$$

$$x(s)(s^2 - 4s) = \frac{-8 - s^4 - 4s^2}{s^2} \quad / : (s^2 - 4s)$$

$$x(s) = \frac{-8 - s^4 - 4s^2}{s(s^2 - 4)} \rightarrow s^2 - 4 = 0$$

$$s^2 = 4 \quad / \sqrt{\ }$$

$$s_{1,2} = \pm\sqrt{4}$$

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^2} + \frac{Ds + E}{s^2 - 4} \quad / : s(s^2 - 4)$$

$$= A(s^2)(s^2 - 4) + B(s)(s^2 - 4) + C(s)(s^2 - 4)$$
$$+ (Ds + E)(s^2)$$

$$= A(s^4 - 4s^2) + B(s^3 - 4s) + C(s^3 - 4s) + (Ds + E)(s^3)$$

$\longrightarrow$

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$$= A S^4 + \underline{4AS^2} + BS^3 + CS^3 + DS^3 + ES^3$$

4)

$$C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \leq 1,\ -1 \leq z < 1\}$$

$$F\begin{pmatrix}(x+2)^2 \\ (y-3)^2\end{pmatrix} \quad \iint (x+2)^2 dy + (y-3)^2 dx$$

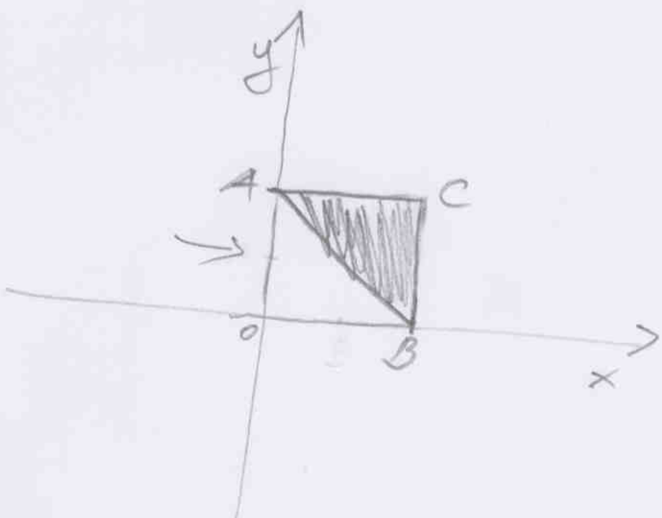
5)

$$\iint_S x + e^y dx dy$$

A(0, 1)

B(1, 0)

C(1, 1)



$$\iint_S x + e^y dx dy$$

$$= \int_0^1 x dx + e^y dy$$