DENIS BLAŠKOV

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoox
IME I PREZIME: **BROJ INDEKSA:**	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S x + e^y \mathrm{d}x\mathrm{d}y$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. — ~~20~~ 15

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 1$ i plohama $z = 1 - y^2$ i $z = x^2 - 1$. — 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 0,\ x'(0) = 4.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \le 1,\ -1 \le z \le 1\}$. Izračunati plošni integral — 20

$$\iint\limits_{\partial C} x\,\mathrm{d}y\mathrm{d}z$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} (3x^2 + y)\,\mathrm{d}x + (3y^2 + x)\,\mathrm{d}y$ — ~~20~~

Ukupno: 35

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

3. $x'''(t) + x'(t) = 0 \quad x(0) = x''(0) = 0 \quad x'(0) = 4$

$s^3F(s) - s^2 f(0) - s f'(0) - f''(0) + sF(s) - f(0)$

$s^3X(s) - 4s + sX(s) = 0$

$X(s)(s^3 + s) - 4s = 0$

$X(s)(s^3+s) = 4s \quad /: (s^3+s)$

$X(s) = \frac{4s}{s^3+s} = \frac{4s}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$

$4s = A(s^2+1) + (Bs+C)\cdot s$

$4s = As^2 + A + Bs^2 + Cs$

$4s = s^2(A+B) + s(C) + A$

$A+B = 0 \to B = -A$

$B = 0$

$C = 4$

$A = 0$

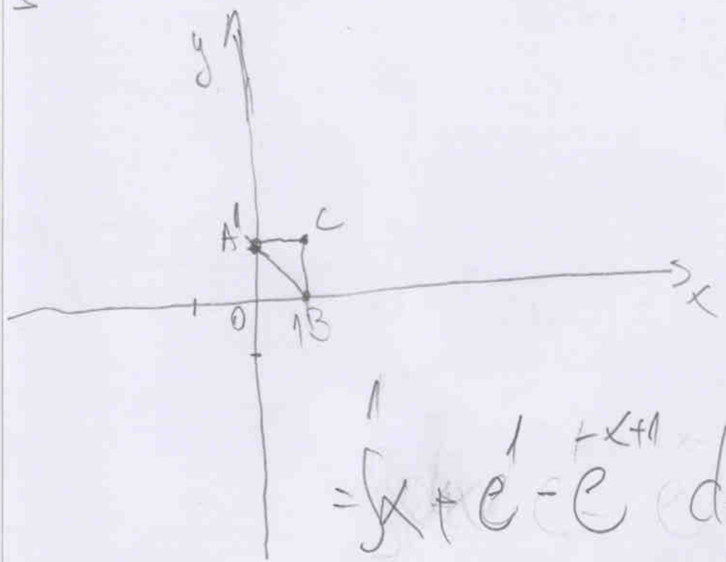
$\mathcal{L}^{-1}X(s) = 0 \cdot \mathcal{L}^{-1}\left(\frac{1}{s}\right) + 4 \cdot \mathcal{L}^{-1}\left(\frac{0 \cdot s + 4}{s^2+1}\right)$

$\mathcal{L}^{-1}X(s) = 0 \cdot 1 + \frac{4}{s^2+1}$

$\mathcal{L}^{-1}X(s) = 4\frac{1}{s^2+1^2} = 4 \cdot \frac{1}{s^2+1^2}$

$x(t) = 4\sin t$ ✓ 20

$\iint\limits_S x+e^y\,dx\,dy$ $A(0,1)$ $B(1,0)$ $C(1,1)$



$\iint\limits_S x+e^y\,dx\,dy = \int_0^1 dx \int_{AB} x+e^y\,dy$

$= \int_0^1 dx \int_{-x+1}^{1} x+e^y\,dy$ ✓

$= \int_0^1 dx\ x+e^y \Big|_{-x+1}^{1}$ 15

$\int x+e^y dy$

$x+\int e^y dy$

$x e^y$

$= \int_0^1 x+e^1-e^{-x+1}\,dx$

$= \frac{x^2}{2}+e+e^{-x+1}\Big|_0^1$

$= \frac{1^2}{2}+e+e^{-1+1}-\left(\frac{0^2}{2}+e+e^{0+1}\right)$

$= \frac{1}{2}+\cancel{e}+1-\cancel{e}-e^1$

$= \frac{3}{2}-e$ ✗

$\int x\,dx = \frac{x^2}{2}$

$\int e\,dx = e$

$\int -e^{-x+1}\,dx$

$-x+1=t$

$-dx=dt$

$dx=-dt$

$-e$

$\int e^t\,d\cancel{t}$

$= e^{-x+1}$

$A(0,1)$

$B(1,0)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1 = \frac{0-1}{1-0}(x-0)$

$y-1 = \frac{-1}{1}(x-0)$

$y-1 = -x$

$y = -x+1$

AC $A(0,1)$

$C(1,1)$

$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1 = \frac{1-1}{1-0}(x-0)$

$y-1 = \frac{0}{1}(x-0)$

$y-1 = 0\cdot(x-0)$

$y=1$

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$$\int_{1,0}^{(e,\pi)} (\underbrace{3x^2+y}_{P})\,dx + (\underbrace{3y^2+x}_{Q})\,dy$$

$$\frac{\partial P}{\partial y} = 6x \qquad \frac{\partial Q}{\partial x} = 6y$$

$$\int_{1,0}^{e,\pi} 6x\,dx + \int_{1,0}^{e,\pi} 6y\,dy$$ ×

$$6 \cdot \frac{x^2}{2}\Big|_{1,0}^{e,\pi} + 6\,\frac{y^2}{2}\Big|_{1,0}^{e,\pi}$$

$$6 \cdot \left(\frac{e^2}{2} - \frac{1}{2}\right) + 6\,\frac{\pi^2}{2}$$

$$= \frac{6e^2}{2} - \frac{6}{2} + \frac{6\pi^2}{2} = \frac{6e^2 - 6 + 6\pi^2}{2}$$