MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa
xxoxo
POPUNJAVA
NASTAVNIK
Broj ↓
bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f'(t) = \cos(t)\,, \quad f(0) = 1,\, f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F}\cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 2 s centrom u točki $T(-1,2,0)$, a koji je orijentiran vanjskom normalom. ~~20~~

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y+2$ i $y=x^2$. ~~20~~ 15

4. Zadana je kruzna uzvojnica (C) s parametrizcijom $t\in[0,2]$: $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Zadano je skalarno polje: $f(x,y,z)=x^2+y^2+z^2$. Izračunati $\int\limits_C f\,\mathrm{d}s$ 20

5. Izracunati $\int_{\widehat{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. ~~20~~

Ukupno: 15

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

1. $f'''(t) - f'(t) = \cos t$ $\qquad f(0)=1$, $f'(0)=f''(0)=0$

$s^3F(t) - s^2\cdot\overset{0}{\cancel{f''(0)}} - s\overset{0\,1}{\cancel{f'(0)}} - \overset{1}{\cancel{f(0)}} \Rightarrow s^3F(t) - 1$

$sF(t) - \overset{1}{\cancel{f(0)}} = sF(t) - 1$ ×

0

$s^3F(t) - sF(t) - 1 + 1 = \frac{s}{s^2+1}$ ×

$F(t)\cdot s(s^2-1) = \frac{s}{s^2+1}$

$F(t) = \frac{\cancel{s}}{\cancel{s}\cdot(s^2-1)\cdot(s^2+1)}$

$F(t) = \frac{1A}{(s^2-1)\cdot(s^2+1)}$

$1 = \frac{As+B}{(s^2-1)} + \frac{Cs+D}{(s^2+1)}$

$1 = (As+B)\cdot(s^2+1) + (Cs+D)\cdot(s^2-1)$

$1 = As^3 + As + Bs^2 + B + Cs^3 - Cs + Ds^2 - D$

s^3 $\quad 0 = A + C$

s^2 $\quad 0 = B + D$

s $\quad 0 = A - C$

$1 = B - D$

$0 = B + D$

$B = 1 + D$

$0 = 1 + D + D$

$2D = -1$

$D = -\frac{1}{2}$ ✓

$1 = B - D$

$1 = B + \frac{1}{2}$

$B = \frac{1}{2}$ ✓

$0 = A + C$

$0 = A - C$

$A = C$

$0 = C + C$

$A = 0$

$C = 0$

$\frac{\overset{0}{A}s + \overset{\frac{1}{2}}{B}}{s^2+1} + \frac{\overset{0}{C}s + \overset{-\frac{1}{2}}{D}}{(s^2+1)}$

$\frac{\frac{1}{2}}{s^2+1} + \frac{-\frac{1}{2}}{s^2+1}$

$\frac{1}{2}\cdot\frac{1}{s^2+1} + -\frac{1}{2}\frac{1}{s^2+1} = \frac{1}{2}\cdot\sin t - \frac{1}{2}\sin t$

$= 0$

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③ $x^2+z^2=1$ $\qquad z=y+2,\ y=x^2=(r\cos\varphi)^2$

$x^2+z^2=r^2$ $\qquad y=z-2=r\sin\varphi-2$

$r=\sqrt{1}=1$ ✓

$$\int_0^{2\pi}d\varphi\int_0^1 r\,dr\int_{r\sin\varphi-2}^{(r\cos\varphi)^2}dy ✓ =\int_0^{2\pi}d\varphi\int_0^1 r\,dr\,\left(y\Big/_{r\sin\varphi-2}^{r^2\cos^2\varphi}\right.=\int_0^{2\pi}d\varphi\int_0^1 r\,dr\,(r^2\cos^2\varphi-r\sin\varphi+2)$$

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$$\int_0^{2\pi}d\varphi\int_0^1 r^3\cos^2\varphi-r^2\sin\varphi+2r$$

$$\int_0^{2\pi}d\varphi\left(\frac{r^4}{4}\cos^2\varphi-\frac{r^3}{3}\sin\varphi+\cancel{2}\frac{r^2}{\cancel{2}}\right)\Big/_0^1$$

$\cos^2\varphi=$

$$\int_0^{2\pi}\left(\frac{1}{4}\cos^2\varphi-\frac{1}{3}\sin\varphi+1\right)d\varphi=$$

$$\int_0^{2\pi}\frac{1}{4}\left(\frac{1+\cos 2\varphi}{2}\right)-\frac{1}{3}\sin\varphi+1)\,d\varphi=\int_0^{2\pi}\left(\frac{1}{8}+\frac{\cos 2\varphi}{8}-\frac{1}{3}\sin\varphi+1\right)d\varphi$$

$$=\frac{1}{8}\int_0^{2\pi}d\varphi ✓+\frac{1}{8}\int_0^{2\pi}\cos 2\varphi\,d\varphi-\frac{1}{3}\int_0^{2\pi}\sin\varphi\,d\varphi+\int_0^{2\pi}d\varphi$$

$2\varphi=t$
$2d\varphi=dt$
$d\varphi=\frac{1}{2}dt$

$\cos t=\sin t$

$$=\frac{1}{8}\cdot 2\pi ✓+\frac{1}{8}\cdot\frac{1}{2}\cdot\overset{0}{\sin 2\pi} ✓+\underbrace{\frac{1}{3}\overset{\boxed{1}\ \times}{\cos 2\pi}}+2\pi ✓$$

$$=\frac{1}{4}\pi ✓+0 ✓+\frac{1}{3}\times+2\pi ✓=\frac{9}{4}\pi+\frac{1}{3}$$

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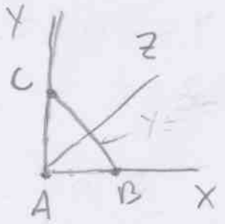
5. A(0,0,0) $\int y\,dx + y\,dy = *$

B(1,0,0)

C(0,1,0)



$$\begin{bmatrix} \cancel{\partial x} \\ \partial y \\ \partial z \end{bmatrix} \times \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 - \partial z y \\ \partial z y - 0 \\ \partial x y - \partial y \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} ✓$$

∂x y

∂y y

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

X X X ∅

0 0

1 0

$$* \int_0^1 \int_0^{1-x} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dx\,dy$$ =?

2. $F = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ $r = 2$

$T(-1,2,0)$

X

4. $t \in [0,2]$ $\begin{bmatrix} x = \cos 2t \\ y = \sin 2t \\ z = t \end{bmatrix}$ - grad $\begin{bmatrix} -\cos 2t \\ = \sin 2t \\ -t \end{bmatrix}$

$$r' = -2(=\sin 2t) = 2 \sin 2t$$
$$-\sin 2t = -2\cos 2t$$
$$-t = -1$$

$$|r'| = \sqrt{(2\sin 2t)^2 + (-2\cos 2t)^2 - 1^2} = \sqrt{4\sin^2 2t + 4\cos 2t + 1}$$

$$= \sqrt{4 \cdot (\underbrace{\sin^2 2t + \cos^2 2t}_{1}) + 1} = \sqrt{5}$$

$$\int_0^2 \sqrt{5}\, dr = \sqrt{5}\, r \Big/_0^2 = 2\sqrt{5}$$