MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa xxoxo

POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 2 s centrom u točki $T(-1,2,0)$, a koji je orijentiran vanjskom normalom. — 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y + 2$ i $y = x^2$. — 20

4. Zadana je kruzna uzvojnica (C) s parametrizcijom $t \in [0,2]$: $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Zadano je skalarno polje: $f(x,y,z) = x^2 + y^2 + z^2$. Izračunati $\int_C f\, ds$ — 20

5. Izracunati $\int_{\widehat{ABC}} y\,dx + y\,dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. — 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

① $f'''(t) - f' = \cos(t), \quad f(0)=1, \; f'(0)=f''(0)=0$

$f''' \rightarrow s^3 y(s) - s^2 \underset{1}{y(0)} - s^1 \underset{0}{y'(0)} - \underset{0}{y''(0)}$

$\rightarrow \boxed{s^3 y(s) - s^2}$

$f' \rightarrow s^1 y(s) - \underset{1}{y(0)} \rightarrow \boxed{s y(s) - 1}$

$s^3 y(s) - s^2 - s y(s) - 1 = \frac{s}{s^2+1}$

$s^3 y(s) - s y(s) = \frac{s}{s^2+1} + s^2 + 1$

$y(s)(s^3 - s) = \frac{s + s^2(s^2+1) + 1(s^2+1)}{s^2+1}$

$y(s)(s^3 - s) = \frac{s + s^4 + s^2 + s^2 + 1}{s^2+1}$

$y(s) = \frac{s^4 + s^2 + s + s^2 + 1}{(s^2+1)(s^3 - s)}$

$y(s) = \frac{s^4 + s^2 + s + s^2 + 1}{(s^2+1)\, s\, (s^2-1)}$

$y(s) = \frac{s^4 + 2s^2 + s + 1}{(s^2-1)\, s\, (s^2+1)}$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$x_{1,2} = \frac{0 \pm \sqrt{-4}}{2}$

$x_{1,2} = \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot (-1)}}{2}$

$= \pm \frac{\sqrt{4}}{2}$

$= \frac{2}{2} = 1$

$s^4 + 2s^2 + s + 1 = \frac{A}{(s^2-1)} + \frac{B}{s} + \frac{Cs + D}{(s^2+1)}$

$= A\, s(s^2+1) + B(s^2-1)(s^2+1) + (Cs + D)(s^2-1)s$

$(s^2-1)\, s\, (s^2+1)$

$= A(s^3 + s) + B(s^4 + s^2 - s^2 - 1) + (Cs + D)(s^3 - s)$

$= As^3 + As + Bs^4 + \cancel{Bs^2} - \cancel{Bs^2} - B + Cs^4 - Cs + Ds^3 - Ds$

$(s^4) \quad 1 = B + C \Rightarrow -C = B - 1$

$C = 1 - B$

$C = 1 + 1$

$\boxed{C = 2}$

$(s^3) \quad 0 = A + D$

$(s^2) \quad 1 = 0$

$(s^1) \quad 1 = A + D - C$

$(s^0) \quad 1 = -B \Rightarrow \boxed{B = -1}$

$1 = A + D - 2$

$-A = -1 + D - 2$

$-A = D - 3$

$A = D + 3$

$0 = A + D$

$0 = D + 3 + D$

$0 = 2D + 3$

$-2D = 3$

$2D = -3$

$\boxed{D = -\frac{3}{2}}$

$A = -\frac{3}{2} + \frac{3}{1}$

$A = \frac{-3 + 6}{2}$

$\boxed{A = \frac{3}{2}}$

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① $f'''(t) - f' = \cos(t)$, $f(0) = 1$, $f'(0) = f''(0) = 0$

$f''' \to s^3 y(s) - s^2 \underset{1}{y(0)} - s^1 \underset{0}{y'(0)} - \underset{0}{y''(0)}$

$\to \boxed{s^3 y(s) - s^2}$

$f' \to s^1 y(s) - y(0) \to \boxed{s y(s) - 1}$

$s^3 y(s) - s^2 - (s y(s) - 1) = \frac{s}{s^2+1}$

$s^3 y(s) - s y(s) = \frac{s}{s^2+1} + s^2 + 1$

$y(s)(s^3 - s) = \frac{s + s^2(s^2+1) + s^2 + 1}{s^2+1}$

$y(s)(s^3 - s) = \frac{s + s^4 + s^2 + s^2 + 1}{s^2+1}$

$y(s)(s^3 - s) = \frac{s^4 + 2s^2 + s + 1}{s^2+1}$

$y(s) = \frac{s^4 + 2s^2 + s + 1}{(s^2+1)(s^3-s)}$

$y(s) = \frac{s^4 + 2s^2 + s + 1}{(s^2-1)\, s\, (s^2+1)}$

$s^4 + 2s^2 + s + 1 = \frac{A}{(s^2-1)} + \frac{B}{s} + \frac{Cs+D}{(s^2+1)} \quad / \cdot (s^2-1)s(s^2+1)$

$= A(s^2+1)s + B(s^2-1)(s^2+1) + (Cs+D)(s^2-1)s$

$= A(s^3+s) + B(s^4 + s^2 - s^2 - 1) + (Cs+D)(s^3-s)$

$= As^3 + As + Bs^4 - B + Cs^4 - Cs^2 + Ds^3 - Ds$

(s^4) $1 = B + C$

(s^3) $0 = A + D$

(s^2) $2 = -C$

(s^1) $1 = A - D$

(s^0) $1 = -B$

$-B = 1$

$B = -1$

$1 = B + C$

$0 = A + D \Rightarrow \cancel{A = D}$

$2 = -C \Rightarrow \boxed{C = -2}$

$1 = A - D$

$1 = -B \Rightarrow \boxed{B = -1}$

B

$$s^4+2s^2+s+1 = \frac{As+B}{(s^2-1)} + \frac{C}{s} + \frac{Ds+E}{(s^2+1)} \Big| \, (s^2-1)\,s\,(s^2+1)$$

$$= (As+B)\,s(s^2+1) + C(s^2-1)(s^2+1) + (Ds+E)(s^2-1)\,s$$

$$= (As+B)(s^3+s) + C(s^4+\cancel{s^2}-\cancel{s^2}-1) + (Ds+E)(s^3-s)$$

$$= As^4 + As^2 + Bs^3 + Bs + Cs^4 - C + Ds^4 - Ds^2 + Es^3 - Es$$

(s^4) $1 = A + C + D$

(s^3) $0 = B + E \Rightarrow -B = E$

(s^2) $2 = A - D$ $\quad \boxed{B = -E} \Rightarrow \boxed{B = \frac{1}{2}}$

(s^1) $1 = B - E \Rightarrow 1 = -E - E$

(s^0) $1 = -C \Rightarrow \boxed{C = -1}$

$1 = -2E$

$-2E = 1$

$2E = -1$

$\boxed{E = -\frac{1}{2}}$

$2 = A - D$

$-A = -D - 2$

$\boxed{A = D + 2}$

$A = D + 2$

$A = 0 + 2$

$\boxed{A = 2}$

$1 = A + C + D$

$1 = D + 2 - 1 + D$

$1 = 2D + 1$

$-2D = 0$

$\boxed{D = 0}$

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① Nastavak

$A = 2,\ B = \frac{1}{2},\ C = -1,\ D = 0,\ E = -\frac{1}{2}$

$$f(t)\ \frac{2s+\frac{1}{2}}{(s^2-1)} - \frac{1}{s} + \frac{0-\frac{1}{2}}{(s^2+1)}$$

$$f(t) = \frac{2s+\frac{1}{2}}{(s^2-1)} - \frac{1}{s} - \frac{\frac{1}{2}}{(s^2+1)}$$

$$\frac{2s}{(s^2-1)} + \frac{\frac{1}{2}}{(s^2-1)} - \frac{1}{s} - \frac{\frac{1}{2}}{(s^2+1)}$$

$\cosh(2t) + \cancel{\sinh(\tfrac{1}{2}t)} - \frac{1}{s}$

$$F(s) = \cosh(2t) + \sinh\left(\tfrac{1}{2}t\right) - 1 - \sin\left(\tfrac{1}{2}t\right)$$

③ $V = ?$

$\cancel{x^2 + z^2 = 1}$

$z = y + 2 \quad y = x^2$

$\cancel{r^2 = 1}$

$\cancel{\boxed{r = 1}}$

$\cancel{p \; q \; k}$

$\cancel{\underbrace{(x-p) + (y-q)}_{r^2} + (z-k) = r^2}$

$\cancel{r^2 + (z-k) = 1}$

$\cancel{r^2 = (-z+k) + 1 \; / \sqrt{}}$

$\cancel{r = \sqrt{(-z+k)+1}}$

③

$F = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix} = -\operatorname{div}$

$F' = F'_x + F'_y + F'_z$

$F' = 0 + 0 + 0$

$F' = 0$

$f = ?$

$f\,d\cancel{y} = -y \,/\int$

$f = -\int y \, dy$

$f = -\frac{y^2}{2} + c(z,1)$?

$f\,dz = -z$

$\left(-\frac{\cancel{y^2}}{\cancel{2}} + c(z,1)\right) dz = -z \,/\int$

$c(z,1)\,dz = -z \,/\int$

$c(z,1) = -\frac{z^2}{2}$

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(2) $\iint_{\partial K} F \cdot ds \qquad F = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix} \qquad r = 2 \qquad T(-1, 2, 0)$

$F' = \sqrt{}$