MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MARINO PRENDŽA **BROJ INDEKSA:** 57659

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

2. Izračunati $\displaystyle\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 2 s centrom u točki $T(-1,2,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y+2$ i $y=x^2$. ~~20~~ 15

4. Zadana je kruzna uzvojnica (C) s parametrizcijom $t\in[0,2]$: $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Zadano je skalarno polje: $f(x,y,z)=x^2+y^2+z^2$. Izračunati $\displaystyle\int_C f\, ds$ ~~20~~ 5

5. Izracunati $\int_{\widehat{ABC}} y dx + y dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2 \ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\, dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\, dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\, d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. $f'''(t) - f'(t) = \cos(t)$

$$s^3F(s) - s^2 f(0) - s\cancel{f'(0)} - \cancel{f''(0)} - (sF(s) - f(0)) = \frac{s}{s^2+1^2}$$

$$s^3F(s) - s^2 - sF(s) + 1 = \frac{s}{s^2+1}$$

$$F(s)(s^3-s) = \frac{s}{s^2+1} + s^2 - 1$$

$$F(s)(s^3-s) = \frac{s^4 + \cancel{s^2} - \cancel{s^2} - 1 + s}{s^2+1}$$

$$F(s)(s^3-s) = \frac{s^4+s-1}{s^2+1} \quad / : (s^3-s)$$

$$F(s) = \frac{s^4+s-1}{(s^2+1)(s^3-s)}$$

$$F(s) = \frac{s^4+s-1}{(s^2+1)s^2(s-1)}$$ X ŠTETA!

$$\frac{s^4+s-1}{(s^2+1)s^2(s-1)} = \frac{As+B}{s^2+1} + \frac{C}{s^2} + \frac{D}{s} + \frac{E}{s-1}$$

$$s^4+s-1 = (As+B)(s^2(s-1)) + C((s^2+1)(s-1)) + Ds(s^2+1)(s-1) + E(s^2(s^2+1))$$

$$s^4+s-1 = As^4 - As^3 + Bs^3 - Bs^2 + Cs^3 - Cs^2 + Cs - C + Ds^4 - Ds^3 + Ds^2 - Ds + Es^4 + Es^2$$

$$s^4+s-1 = s^4(A+D+E) + s^3(-A+B+C-D) + s^2(-B-C+D+E) + s(C-D) - C$$

$A+D+E=1 \longrightarrow A-2+E=1$

$-A+B+C-D=0$

$-B-C+D+E=0$

$C-D=1 \longrightarrow -1-D=1$

$\boxed{C=-1}$

$-D=1+1$

$-D=2 \;/:(-1)$

$\boxed{D=-2}$

$A=1+2-E$

$A=3-E$

$A=3-\frac{7}{2}$

$\boxed{A=-\frac{1}{2}}$

$$\begin{aligned} -A+B+C-D&=0 \\ -B-C+D+E&=0 \\ \hline -3+E+B-1-2&=0 \end{aligned}$$

$-B-(-1)-2+E=0$

$-B+1-2+E=0$

$-B+E=1$

$$+\begin{cases} E+B=6 \\ E-B=1 \end{cases}$$

$2E=7 \;/:2$

$\boxed{E=\frac{7}{2}}$

$-B+\frac{7}{2}=1$

$-B=1-\frac{7}{2}$

$-B=-\frac{5}{2} \;/:(-1)$

$\boxed{B=\frac{5}{2}}$

$$F(s)=-\frac{1}{2}\cdot\frac{s}{s^2+1}+\frac{5}{2}\cdot\frac{1}{s^2+1}-\frac{1}{s^2}-\frac{2}{s}+\frac{7}{2}\cdot\frac{1}{s-1}$$

$$F(t)=\mathcal{L}^{-1}\left\{-\frac{1}{2}\cdot\frac{s}{s^2+1}+\frac{5}{2}\cdot\frac{1}{s^2+1}-\frac{1}{s^2}-\frac{2}{s}+\frac{7}{2}\cdot\frac{1}{s-1}\right\}$$

$$F(t)=-\frac{1}{2}\cos(t)+\frac{5}{2}\sin(t)-t-2+\frac{7}{2}e^t$$

LAKO SE PROVJERI DA NE ZADOVOLJAVA

$f'(0)=0$

MARINO PRENDŽA 57659

4. $t \in [0, 2]$

$x = \cos 2t$

$y = \sin 2t$

$z = t$

$$r'(t) = \begin{bmatrix} -\sin 2t \\ \cos 2t \\ 1 \end{bmatrix}$$

$f(x, y, z) = x^2 + y^2 + z^2$

$\int_C f\, ds = ?$

$\|r'(t)\| = \sqrt{(-\sin 2t)^2 + (\cos 2t)^2 + 1^2}$

$\|r'(t)\| = \sqrt{\sin^2 4t + \cos^2 4t + 1}$

$\|r'(t)\| = \sqrt{4(\underbrace{\sin^2 t + \cos^2 t}_{1}) + 1}$

$\|r'(t)\| = \sqrt{4+1}$

$\|r'(t)\| = \sqrt{5}$ ✓

$f \circ r = x^2 + y^2 + z^2$

$f \circ r = (\cos 2t)^2 + (\sin 2t)^2 + t^2$

$f \circ r = \cos^2 4t + \sin^2 4t + t^2$ ✓

$f \circ r = 4(\underbrace{\cos^2 t + \sin^2 t}) + t^2$ ✗

$f \circ r = 4 + t^2$

$\int_0^2 f \circ r \cdot \|r'(t)\|$ ✓

VIDIM DA STE NAUČILI. ŠTETA ZBOG GREŠKE.

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$\int_0^2 (4 + t^2)\sqrt{5}\, ds$ ✗ $= \int_0^2 (4\sqrt{5} + \sqrt{5}t^2)\, ds$

$= 4\sqrt{5} \cdot 2 + \sqrt{5}\frac{2^3}{3} = 8\sqrt{5} + \frac{8}{3}\sqrt{5} =$

$= \frac{32}{3}\sqrt{5}$

3. $x^2 + z^2 = 1$

$r^2 = 1 /\sqrt{\ }$

$r = 1$

$r \in [0, 1]$

$\varphi \in [0, 2\pi]$

$y \in [r\sin\varphi - 2, r^2\cos^2\varphi]$

$z = y + 2$

$-y = -z + 2$

$y = z - 2$

$y = r\sin\varphi - 2$

$y = x^2$

$y = (r\cos\varphi)^2$

$y = r^2\cos^2\varphi$

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$$V = \int_0^{2\pi} d\varphi \int_0^1 r\, dr \int_{r\sin\varphi - 2}^{r^2\cos^2\varphi} dy = \int_0^{2\pi} d\varphi \int_0^1 r(r^2\cos^2\varphi - (r\sin\varphi - 2))\, dr =$$ ✓

$$\int_0^{2\pi} d\varphi \int_0^1 r^3\cos^2\varphi - r^2\sin\varphi + 2r\,dr = \int_0^{2\pi} d\varphi \int_0^1 r^3\cos^2\varphi\, dr - \int_0^{2\pi} d\varphi \int_0^1 r^2\sin\varphi + \int_0^{2\pi} d\varphi\, 2\int_0^1 r\,dr$$

$$= \int_0^{2\pi} d\varphi \left(\frac{r^4}{4}\right)\cos^2\varphi\Big|_0^1 - \int_0^{2\pi} d\varphi \left(\frac{r^3}{3}\right)\sin\varphi\Big|_0^1 + \int_0^{2\pi} d\varphi \left(\cancel{2}\cdot\frac{r^2}{\cancel{2}}\right)\Big|_0^1$$

$$= \frac{1}{4}\int_0^{2\pi}\cos^2\varphi\, d\varphi - \frac{1}{3}\int_0^{2\pi}\sin\varphi\, d\varphi + \int_0^{2\pi} d\varphi = \frac{1}{4}\underbrace{\int_0^{2\pi}\frac{1+\cos(2\varphi)}{2}\, d\varphi}_{} - \frac{1}{3}\int_0^{2\pi}\sin\varphi\, d\varphi + \int_0^{2\pi} d\varphi =$$

$$= \underbrace{\int_0^{2\pi}\frac{1}{2}\, d\varphi}_{=\pi} + \frac{1}{2}\underbrace{\int_0^{2\pi}\cos(2\varphi)\, d\varphi}_{=0}$$

$$= \frac{1}{4}\cdot\frac{1}{2}\int_0^{2\pi}\cos(2\varphi)\, d\varphi - \frac{1}{3}\int_0^{2\pi}\sin\varphi\, d\varphi + \int_0^{2\pi} d\varphi$$

$$= \frac{1}{8}\int_0^{2\pi}\cos(2\varphi)\, d\varphi - \frac{1}{3}\int_0^{2\pi}\sin\varphi\, d\varphi + \int_0^{2\pi} d\varphi =$$

$$= \frac{1}{8}\int_0^{2\pi}\frac{1}{2}\cos\varphi\, d\varphi - \frac{1}{3}\int_0^{2\pi}\sin\varphi\, d\varphi + \int_0^{2\pi} d\varphi$$

$\cancel{\cos}(2\varphi) = t$ $\quad d\varphi = \frac{1}{2}\, dt$

$2\varphi\, d\varphi = dt \;/:2$

$$= \frac{1}{16}\sin\varphi\Big|_0^{2\pi} + \frac{1}{3}\cos\varphi\Big|_0^{2\pi} + \varphi\Big|_0^{2\pi}$$

$$= \frac{1}{16}(\underbrace{\sin 2\pi - \sin 0}_{0}) + \frac{1}{3}(\underbrace{\cos 2\pi - \cos 0}_{1-1=0}) + 2\pi$$

$$= 2\pi$$