**MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANĐELO UGRINIĆ BROJ INDEKSA: 55581

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu:
$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$
20

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i $\partial K$ rub kugle $K$ radijusa 2 s centrom u točki $T(-1,2,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y + 2$ i $y = x^2$. 20

4. Zadana je kruzna uzvojnica $(C)$ s parametrizcijom $t \in [0,2]$: $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Zadano je skalarno polje: $f(x,y,z) = x^2 + y^2 + z^2$. Izračunati $\int_C f\,\mathrm{d}s$ 20

5. Izracunati $\int_{\widehat{ABC}} y\mathrm{d}x + y\mathrm{d}y$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha $A$ preko $B$ i $C$ do ponovo vrha $A$. Koristiti Stokesovu formulu. 20

Ukupno:

Tablica integrala

| | | |
|---|---|---|
| $\int \mathrm{d}x = x + C$ | $\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$ | $\int \frac{\mathrm{d}x}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$ |
| $\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$ | $\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$ | $\int \frac{\mathrm{d}x}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$ |
| $\int \frac{\mathrm{d}x}{x} = \ln\lvert x\rvert + C$ | $\int \sinh x\,\mathrm{d}x = \cosh x + C$ | $\int \frac{\mathrm{d}x}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$ |
| $\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$ | $\int \cosh x\,\mathrm{d}x = \sinh x + C$ | $\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$ |
| $\int \sin x\,\mathrm{d}x = -\cos x + C$ | $\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$ | $\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$ |
| $\int \cos x\,\mathrm{d}x = \sin x + C$ | $\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$ | $\int \frac{\mathrm{d}x}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$ |
| $\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$ | $\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$ | $\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$ |
| $\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$ | $\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$ | $\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$ |

Tablica Laplaceovih transformacija:

| $f(t)$ | $F(s) = \mathcal{L}[f](s)$ | $f(t)$ | $F(s) = \mathcal{L}[f](s)$ |
|---|---|---|---|
| $1$ | $\frac{1}{s}$ | $\sinh(at)$ | $\frac{a}{s^2 - a^2}$ |
| $c$ | $\frac{c}{s}$ | $\cosh(at)$ | $\frac{s}{s^2 - a^2}$ |
| $t$ | $\frac{1}{s^2}$ | $e^{-at} f(t)$ | $F(s + a)$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ | $f(at)$ | $\frac{1}{a}F(\frac{s}{a})$ |
| $\frac{1}{\sqrt{\pi t}}$ | $\frac{1}{\sqrt{s}}$ | $t^n f(t)$ | $(-1)^n F^{(n)}(s)$ |
| $e^{-at}$ | $\frac{1}{s+a}$ | $\frac{f(t)}{t}$ | $\int_s^\infty F(q)\,\mathrm{d}q$ |
| $t\,e^{-at}$ | $\frac{1}{(s+a)^2}$ | $\int_0^t f(\tau)\,\mathrm{d}\tau$ | $\frac{F(s)}{s}$ |
| $(1 - at)\,e^{-at}$ | $\frac{s}{(s+a)^2}$ | $f'(t)$ | $sF(s) - f(0)$ |
| $\sin(at)$ | $\frac{a}{s^2 + a^2}$ | $f''(t)$ | $s^2F(s) - sf(0) - f'(0)$ |
| $\cos(at)$ | $\frac{s}{s^2 + a^2}$ | $f'''(t)$ | $s^3F(s) - s^2f(0) - sf'(0) - f''(0)$ |

ANĐELO UGRINIĆ

① $f'''(t) - f'(t) = \cos(t)$ $\quad f(0)=1$

$f'(0)=f''(0)=0$

$s^3F(s) - s^2f(0) - sf'(0) - f''(0) - (sF(s) - f(0)) = \frac{s}{s^2+1^2}$

$s^3F(s) - s^2 - sF(s) + 1 = \frac{s}{s^2+1}$

$F(s)(s^3-s) = \frac{s}{s^2+1} + s^2 - 1$

$F(s)(s^3-s) = \frac{s+s^4+s^2-s^2-1}{s^2+1} = \frac{s^4+s-1}{s^2+1}$

$F(s) = \frac{s^4+s-1}{s^2+1} \cdot (s^3-s)$ $\quad (s^3-s) = s(s^2-1)$

$F(s) = \frac{s^4+s-1}{s(s^2+1)(s^2-1)}$

$F(s) =$

$\frac{1}{\quad} = \frac{A}{\quad} + \frac{B}{\quad} + \frac{C}{\quad}$

$s^4+s-1 = \frac{A}{s} + \frac{Bs+C}{s^2+1} + \frac{Ds+E}{s^2-1}$ ?

$=$

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④

$t \in [0, 2]$

$x = \cos 2t$

$y = \sin 2t$

$z = t$

$x \in [0, 2]$

$y \in [0, 2\pi]$

$x = r\cos\varphi$

$y = r\sin\varphi$

$r' = \begin{matrix} -\sin 2t \\ \cos 2t \\ 1 \end{matrix}$

$\|r'\| = \sqrt{(-\sin 2t)^2 + (\cos 2t)^2 + 1^2}$

$= \sqrt{4\sin^2 t + 4\cos^2 t) + 1}$

$= \sqrt{4(\sin^2 t + \cos^2 t) + 1}$

$= \sqrt{4 + 1}$

$= \sqrt{5}$

KAKAV DVOSTRUKI INTEGRAL

$$\int_0^2 \int_0^{2\pi} \sqrt{5}\, dx\, dy = \sqrt{5} \int_0^2 \int_0^{2\pi} dx\, dy = \sqrt{5} \int_0^2 y \Big|_0^{2\pi} dx = \sqrt{5} \int_0^2 2\pi\, dx = \sqrt{5}\, 2\pi\, x \Big|_0^2$$

$= 2\sqrt{5}\pi 2 = \underline{4\sqrt{5}\pi}$

TRAŽI SE $\int_C f\, ds = ?$

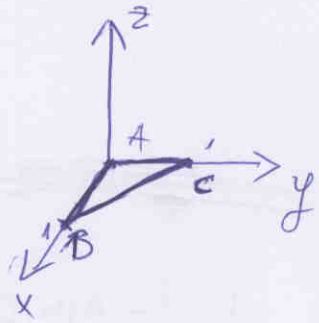
(5) ANDELO UGRINIĆ

$$\int_{\vec{ABC}} y\,dx + y\,dy$$


$A(0,0,0)$
$B(1,0,0)$
$C(0,1,0)$

$$\begin{bmatrix} y \\ y \\ 0 \end{bmatrix} \qquad \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} \begin{bmatrix} 0 - y\,dz \\ y\,dz - 0 \\ y\,dx - y\,dy \end{bmatrix}$$

$$\begin{bmatrix} dx \\ dy \end{bmatrix} \begin{bmatrix} y \\ y \end{bmatrix}$$




$$= \begin{bmatrix} 0-0 \\ 0-0 \\ 0-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-0 \\ 0-0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$0 \times 0$$
$$1 \times 0$$

$$\int_{\vec{ATC}} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} dx\,dy = -\int_{\vec{APC}} dx\,dy$$