1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

2. Izračunati $\iint\limits_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 2 s centrom u točki $T(-1, 2, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y + 2$ i $y = x^2$. 20

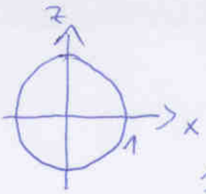
4. Zadana je kruzna uzvojnica (C) s parametrizcijom $t \in [0, 2]$: $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izračunati $\int\limits_C f\, ds$ 20

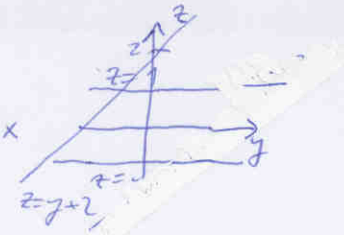
5. Izracunati $\int_{\widehat{ABC}} y dx + y dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0, 0, 0)$, $B(1, 0, 0)$, $C(0, 1, 0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

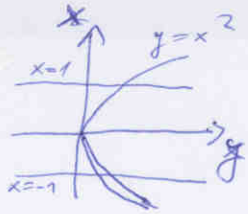
(3) VALJAK $x^2+z^2=1$

RAVNINA $z=y+2$

PARABOLA $y=x^2$







$$V=\int_0^{2\pi}\int_0^1 r\int_{r\sin\varphi-2}^{r^2\cos^2\varphi} 1\,dy\,dr\,d\varphi = \dots = \frac{9}{4}\pi$$

VIDI PRENDŽA

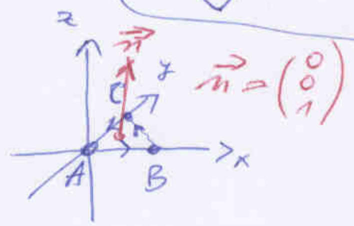
(3D)

---

(5)

$$\oint_{\overrightarrow{ABC}} \begin{pmatrix} y\\ y\\ 0\end{pmatrix} = \iint_{ABC} \underbrace{rot\begin{pmatrix}y\\ y\\ 0\end{pmatrix}}_{=\begin{pmatrix}0\\ 0\\ -1\end{pmatrix}} = \iint_{ABC}\begin{pmatrix}0\\ 0\\ -1\end{pmatrix}\cdot\vec{n}\,dS$$

$$= \int_0^1\int_0^{1-x} -1\,dy\,dx = -\frac{1}{2}$$



$\vec{n}=\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}$

---

(4) $C \dots \quad t\in[0,2] \quad \begin{cases} x=\cos 2t\\ y=\sin 2t\\ z=t\end{cases}$

$f(x,y,z)=x^2+y^2+z^2$

$$\int_C f\,ds = \int_0^2 (\cos^2 2t+\sin^2 2t+t^2)\cdot\sqrt{4\sin^2(2t)+4\cos^2(2t)+1}\,dt = \int_0^2 (1+t^2)\cdot\sqrt{5}\,dt = \dots$$

$\boxed{\begin{aligned} x'&=-2\sin 2t\\ y'&=2\cos 2t\\ z'&=1\end{aligned}}$

---

(1) $f'''(t)-f'(t)=\cos(t)$

$f(0)=1,\ f'(0)=0,\ f''(0)=0$ $\xrightarrow[\text{TRANSFORMACIJA}]{\text{LAPLACEOVA}}$

$$s^3F(s)-s^2-sF(s)+1=\frac{s}{s^2+1}$$

$$F(s)=\frac{s^4+s^2-1}{s(s-1)(s+1)(s^2+1)} = \frac{A}{s}+\frac{B}{s-1}+\frac{C}{s+1}+\frac{Ds+E}{s^2+1}$$

$$s^4+s^2-1 = A(s-1)(s+1)(s^2+1)+Bs(s+1)(s^2+1)+Cs(s-1)(s^2+1)+(Ds+E)s(s-1)(s+1)$$

UVRSTITI: $s=0 \Rightarrow -1=A\cdot(-1)\cdot 1\cdot 1 \Rightarrow A=1$

$s=1 \Rightarrow 1=B\cdot 1\cdot 2\cdot 2 \Rightarrow B=\frac{1}{4}$

$s=-1 \Rightarrow 1=C\cdot(-1)\cdot(-2)\cdot 2 \Rightarrow C=\frac{1}{4}$

$s=i \Rightarrow -1=\underbrace{(Di+E)i(i-1)(i+1)}_{(Di+E)i(-2)}$

$\Rightarrow Di+E=\frac{1}{2i}=-\frac{i}{2}$ (1)

$s=-i \Rightarrow -1=(-Di+E)(-i)\underbrace{(-i-1)(-i+1)}_{(-2)}$

$\Rightarrow -Di+E=\frac{i}{2}$ (2)

$2E=0$ (1)+(2)

$\Rightarrow E=0 \Rightarrow D=-\frac{1}{2}$

$$F(s)=\frac{1}{s}+\frac{1}{4}\cdot\frac{1}{s-1}+\frac{1}{4}\cdot\frac{1}{s+1}+\left(-\frac{1}{2}\right)\frac{s}{s^2+1}$$

INVERZNI LAPLACE

LAKO SE PROVJERI DA ZADOVOLJAVA POLAZNE UVJETE

$$f(t)=1+\frac{1}{4}e^{t}+\frac{1}{4}e^{-t}-\frac{1}{2}\cos t$$

---

(2)

$$\iint_{\partial K}\begin{pmatrix}y\\ z\\ 1\end{pmatrix}\cdot dS = \iiint_K \underbrace{div\begin{pmatrix}y\\ z\\ 1\end{pmatrix}}_{=0} = 0$$