MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANTE ŠUŠNJARA BROJ INDEKSA:

Grupa xxoxo
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F}\cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i ∂K rub kugle K radijusa 2 s centrom u točki $T(-1,2,0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2+z^2=1$ i ravninama $z=y+2$ i $y=x^2$. 20

4. Zadana je kruzna uzvojnica (C) s parametrizcijom $t \in [0,2]$: $x=\cos 2t$, $y=\sin 2t$ i $z=t$. Zadano je skalarno polje: $f(x,y,z)=x^2+y^2+z^2$. Izračunati $\int_C f\,ds$ 20

5. Izracunati $\int_{\widehat{ABC}} y\,dx + y\,dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0,0,0)$, $B(1,0,0)$, $C(0,1,0)$ usmjerena redom od vrha A preko B i C do ponovo vrha A. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

1) $f'''(t) - f'(t) = \cos(t)$ $f'(0)\ f''(0) = 0$, $f(0) = 1$

$f'''(t) = s^3\mathcal{L}(s) - s^2 \overset{1}{f(0)} - s\overset{0}{f'(0)} - \overset{0}{f''(0)} = s^3\mathcal{L}(s) - s^2$

$f'(t) = s\mathcal{L}(s) - 1$

$\cos(t) = \frac{s}{s^2+a^2}$

$s^3\mathcal{L}(s) - s^2 - s\mathcal{L}(s) - 1 = \frac{s}{s^2+a^2}$

$s^3\mathcal{L}(s) - s\mathcal{L}(s) = \frac{s}{s^2+a} + s^2 + 1$

$\mathcal{L}(s)(s^3 - s) =$

$\mathcal{L}(s) = \frac{s + s^2(s^3-s) + (s^3-s)}{(s^2+a)(s^3-s)} = \cancel{s + s^5 - s^3 + s^3 - s}$

$\mathcal{L}(s) =$

$\mathcal{L}(s) = \frac{s^5}{(s^2+a)(s^3-s)} = \frac{\cancel{s^5}}{\cancel{s^5} - s^3 + as^3 - as}$

$= \frac{1}{-s^3 + as^3 - as}$

$= \frac{1}{-s^3 + a(s^3 - s)}$

$s^2 L(s) - s^2 L(s) = \frac{s}{s^2+a^2} + s^2 - 1$

s^2

$L(s)(s^2 - s) = \frac{s}{s^2+a^2} + s^2 - 1 \,/\, (s^2 - s)$

$L(s) = \frac{s + s^2(s^2 - s) + \cdot (s^2 - s)}{(s^2+a^2)(s^2 - s)}$

$= \frac{}{s(s+a^2)(s^2-s)s}$

ANTE
ŠUŠNJARA