| | |
|---|---|
| **MATEMATIKA 3**: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**<br>**IME I PREZIME:** IVA PEZEROVIĆ **BROJ INDEKSA:** | Grupa xxoxo POPUNJAVA NASTAVNIK Broj ↓ bodova |

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$f'''(t) - f'(t) = \cos(t), \quad f(0) = 1,\ f'(0) = f''(0) = 0.$$

2. Izračunati $\iint_{\partial K} \mathbf{F} \cdot d\mathbf{S}$ gdje je $\mathbf{F} = \begin{pmatrix} y \\ z \\ 1 \end{pmatrix}$ i $\partial K$ rub kugle $K$ radijusa 2 s centrom u točki $T(-1, 2, 0)$, a koji je orijentiran vanjskom normalom. 20

3. Izracunati volumen tijela omeđenog valjkom $x^2 + z^2 = 1$ i ravninama $z = y + 2$ i $y = x^2$. 20

4. Zadana je kruzna uzvojnica $(C)$ s parametrizcijom $t \in [0, 2]$: $x = \cos 2t$, $y = \sin 2t$ i $z = t$. Zadano je skalarno polje: $f(x, y, z) = x^2 + y^2 + z^2$. Izračunati $\int_C f\, ds$ 20

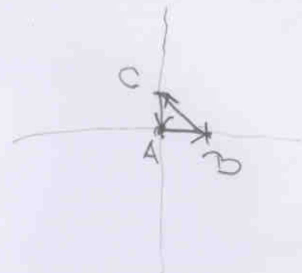
5. Izracunati $\int_{\widehat{ABC}} y dx + y dy$ gdje je $\widehat{ABC}$ krivulja koja ide bridovima trokuta s vrhovima $A(0, 0, 0)$, $B(1, 0, 0)$, $C(0, 1, 0)$ usmjerena redom od vrha $A$ preko $B$ i $C$ do ponovo vrha $A$. Koristiti Stokesovu formulu. 20

Ukupno: 0

Tablica integrala

| | | |
|---|---|---|
| $\int dx = x + C$ | $\int \frac{dx}{\cos^2 x} = \tan x + C$ | $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$ |
| $\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$ | $\int \frac{dx}{\sin^2 x} = -\cot x + C$ | $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$ |
| $\int \frac{dx}{x} = \ln\lvert x\rvert + C$ | $\int \sinh x\, dx = \cosh x + C$ | $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$ |
| $\int a^x dx = \frac{a^x}{\ln a} + C$ | $\int \cosh x\, dx = \sinh x + C$ | $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$ |
| $\int \sin x\, dx = -\cos x + C$ | $\int \tanh x\, dx = \ln\lvert\cosh x\rvert$ | $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$ |
| $\int \cos x\, dx = \sin x + C$ | $\int \coth x\, dx = \ln\lvert\sinh x\rvert$ | $\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$ |
| $\int \tan x\, dx = -\ln\lvert\cos x\rvert$ | $\int \frac{dx}{\cosh^2 x} = \tanh x + C$ | $\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$ |
| $\int \cot x\, dx = \ln\lvert\sin x\rvert$ | $\int \frac{dx}{\sinh^2 x} = -\coth x + C$ | $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$ |

$$\begin{array}{cccc} & x & y & z \\ A & (0, & 0, & 0) \\ B & (1, & 0, & 0) \\ C & (0, & 1, & 0) \end{array}$$



Tablica Laplaceovih transformacija:

| $f(t)$ | $F(s) = \mathcal{L}[f](s)$ | $f(t)$ | $F(s) = \mathcal{L}[f](s)$ |
|---|---|---|---|
| $1$ | $\frac{1}{s}$ | $\sinh(at)$ | $\frac{a}{s^2 - a^2}$ |
| $c$ | $\frac{c}{s}$ | $\cosh(at)$ | $\frac{s}{s^2 - a^2}$ |
| $t$ | $\frac{1}{s^2}$ | $e^{-at} f(t)$ | $F(s + a)$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ | $f(a t)$ | $\frac{1}{a}F(\frac{s}{a})$ |
| $\frac{1}{\sqrt{\pi t}}$ | $\frac{1}{\sqrt{s}}$ | $t^n f(t)$ | $(-1)^n F^{(n)}(s)$ |
| $e^{-at}$ | $\frac{1}{s+a}$ | $\frac{f(t)}{t}$ | $\int_s^\infty F(q)\, dq$ |
| $t\, e^{-at}$ | $\frac{1}{(s+a)^2}$ | $\int_0^t f(\tau)\, d\tau$ | $\frac{F(s)}{s}$ |
| $(1 - at)\, e^{-at}$ | $\frac{s}{(s+a)^2}$ | $f'(t)$ | $sF(s) - f(0)$ |
| $\sin(at)$ | $\frac{a}{s^2 + a^2}$ | $f''(t)$ | $s^2F(s) - sf(0) - f'(0)$ |
| $\cos(at)$ | $\frac{s}{s^2 + a^2}$ | $f'''(t)$ | $s^3F(s) - s^2f(0) - sf'(0) - f''(0)$ |

1. $(s^3F(s) - s^2f(0) - sf'(0) - f''(0)) - (sF(s) - f(0)) = \cos(t)$

$(s^3F(s) - s^2(1) - s(0) - (0)) - (sF(s) - 1) = \frac{s}{s^2 + a^2}$

$(s^3F(s) - 1s^2) - (sF(s) - 1) = \frac{s}{s^2 + a^2}$

$s^3 - 1s^2 - s + 1 = \frac{s}{s^2}$

$s^5 - s^4 - s^3 + s^2 = s$

$s^5 - s^3 = s \qquad -s^4 + s^2 = s$

$s^{-15} = s^1 \qquad -s^{-8} = s^1$

5. (AB) $y - y_1 = \frac{y_2 - y_1}{x_2 - y_1}(x - x_1)$

$y - 0 = \frac{0 - 0}{0 - 1}(x - 0)$

$y = x$

NALAŽENJE PRAVACA SE NE BODUJE ~~ZASEBNO~~

BC

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \ast (x - x_1)$$

$$y - 0 = \frac{1 - 0}{0 - 1} \ast (x - 1)$$

$$y = x - 1$$

CA

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \ast (x - x_1)$$

$$y - 1 = \frac{0 - 1}{0 - 0} \ast (x - 0)$$

$$y = x + 1$$

$$\int_{\underset{x-1}{\widehat{ABC}}} y\,dx + \underset{x+1}{y}\,dy$$

$$\int_{x}^{x-1} dx\,dy + \int_{x-1} dy\,dx$$

$$\int$$

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**IME I PREZIME:** BRUNO LIPOTICA **BROJ INDEKSA:** 54960

Grupa xxoxo

POPUNJAVA NASTAVNIK Broj ↓ bodova

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3

$x^2+z^2=1$

$z = y+2$

$y = x^2$

$\varphi(0, 2\pi)$

$r(0, 1)$

$r^2\cos^2\varphi$

$r^2\sin^2\varphi$

$2\,(r\sin\varphi+2,$

$r^2\cos^2\varphi$

$\int_0^{2\pi} d\varphi \int_0^1 r\,dr \int dz$

$r\sin\varphi+2$

$\int_0^{2\pi} d\varphi \int_0^1 r\,(r^2\cos^2\varphi - r$

1) $f'''(t) - f'(t) = \cos(t)$ $\qquad f(0) = 1$

$\qquad f'(0) = f''(0) = 0$

$s^3F(s) - s^2 f(0) - sf'(0) - f''(0) - sF(s) - f(0) = \frac{s}{s^2-1}$

$s^3F(s) - s^2(1) - sF(s) - 1 = \frac{s}{s^2-1}$

$s^3F(s) + sF(s) =$

$s^3F(s) + sF(s) = \frac{s^2 - s + 1}{s^2-1}$

$F(s)(s^3+s) = s^2 - s + 1 \quad / s^2-1)$

$= s^2(s^2-1) - s + s^2 - 1)$

$= s^4 - s^2 + s + s^2 - 1$

$F(s)(s^3+s) = s^4 - s^2 + s + s^2 - 1$

$F(s)(s^3+s) = s^4 + s - 1 \quad / s^3 + s \quad s(s^2+1)$

$\frac{s^4+s-1}{s(s^2+1)} =$

$\frac{s^4+s-1}{s(s^2+1)} = \frac{A}{s} + \frac{B+C}{s^2+1}$

BRUNO CIPOTICA

(3) $x^2 + z^2 = 1 \qquad z = y + 2 \qquad y = x^2$

$$\int_0^{2\pi} d\varphi \int_0^1 r\,dr \int_{r\sin\varphi + 2}^{r^2\cos^2\varphi} dz$$ ✗ ✗

$\varphi (0, 2\pi)$

$r (0, 1)$

$z (r \sin\varphi + 2, r^2 \cos^2\varphi)$ ✗

$$\int_0^{2\pi} d\varphi \int_0^1 r\,(r^2\cos^2\varphi - r\sin\varphi + 2)$$

$$\int_0^{2\pi} d\varphi \int_0^1 r^2\cos\varphi - r^2\sin\varphi + 2r$$

$$\int_0^{2\pi} d\varphi \; \frac{r^3}{3}\cos\varphi - \frac{r^3}{3}\sin\varphi + \cancel{2}\frac{r^2}{\cancel{2}}\Big|_0^1$$

$$\int_0^{2\pi} d\varphi \; \frac{1}{3}\cos\varphi - \frac{1}{3}\sin\varphi + 1$$

$$\frac{1}{3}\sin\varphi + \frac{1}{3}\cos\varphi + 1 \Big|_0^{2\pi}$$

$$\frac{1}{3}\sin 2\pi + \frac{1}{3}\cos 2\pi + 1 - \frac{1}{3}\sin 0 + \frac{1}{3}\cos 0 + 1$$

$$0 + \frac{1}{3}\cdot 1 + 1 - \frac{1}{3} - 1$$

$$= \frac{1}{3} + 1 + \frac{2}{3}$$

$$= \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

NASTAVAK PRVOG ZADATKA BRUNO LIPOTICA

$$A(s^2+1) + B+C(s)$$

$$As^2 + A + Bs + Cs$$