**MATEMATIKA 2**

14. lipnja 2012.

**Ime i prezime:** LUKA KURILIĆ **Broj indeksa:** 58076

**Vrijeme: od** 09h 05min **do** 10:21 **♣3** **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *(15)* Integriraj
$$\int_1^2 \frac{\sin(\ln x)}{x}\,dx$$

2. *(20)* Integriraj
$$\int \frac{x}{(x-1)(x^2+x+1)}\,dx$$

3. *(20)* Odredi površinu koju zatvaraju krivulja $x = y^2 - 2y + 2$ i pravac $2x + y = 9$.

4. *(10+10)*

a) Ispitaj ekstreme funkcije
$$f(x,y) = x^3 + y^3 - 15xy$$

b) Odredi domenu funkcije:
$$f(x,y) = \arcsin(x+y)$$

5. *(10+15)* Riješi sljedeće diferencijalne jednadžbe:

a)
$$xy' - 4y = x^3$$

b)
$$y'' + 9y = 12e^{-3x}$$

① $\int_1^2 \frac{\sin(\ln x)}{x}\,dx = \int_1^2 \sin\frac{1}{x}\,dx$ ✗ $= \int_1^2 \left(\frac{\sin}{x}\right)\,dx = \int_1^2 \sin x^2\,dx$

$= [\int \sin 2^2\,dx - \int \sin 1^2\,dx]$ $= [8\sin 4\,dx - \sin dx]$

$= [4\int \sin dx - \int \sin dx] = [-4\cos + \cos]$

$= 0{,}9975\ldots + 0{,}9998\ldots$

$= 1{,}9974\ldots$ ⇒ NASTAVAK

0



VIDI RJEŠENJE 3

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME

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(3) $x = y^2 - 2y + 2$ $\quad 2x + y = 9$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{2 \pm \sqrt{4-4\cdot1\cdot2}}{2\cdot1}$

$x_{1,2} = \frac{2 \pm \sqrt{4-8}}{2}$

$x_{1,2} = \frac{2 \pm \sqrt{-4}}{2}$

$T\left(-\frac{b}{2a} \mid \frac{4ac-b^2}{4a}\right)$

$T\left(+\frac{+1}{2\cdot1} \mid \frac{4\cdot1\cdot(-7)+1}{4}\right)$

$T\left(\frac{1}{2} \mid \frac{-27}{4}\right)$

$T\left(\frac{-b}{2a} \mid \frac{4ac-b^2}{4a}\right)$

$T\left(\frac{2}{2} \mid \frac{4\cdot1\cdot2-4}{4}\right)$

$T\left(1 \mid \frac{8-4}{4}\right)$

$T(1 \mid -1)$

$x = y^2 - 2y + 2 \Rightarrow x - y^2 + 2y - 2 = 0$

$2x + y = 9 \qquad 2x + y - 9 = 0$

~~$x - y^2 + 2y - 2 - 2x - y + 9$~~

$x - y^2 + 2y - 2 = 2x + y - 9$

$x - 2x - y^2 + 2y - y - 2 + 9 = 0$

$-y^2 - x + y + 7 = 0$

$x = y^2 - y - 7$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{1 \pm \sqrt{1-4\cdot1\cdot(-7)}}{2}$

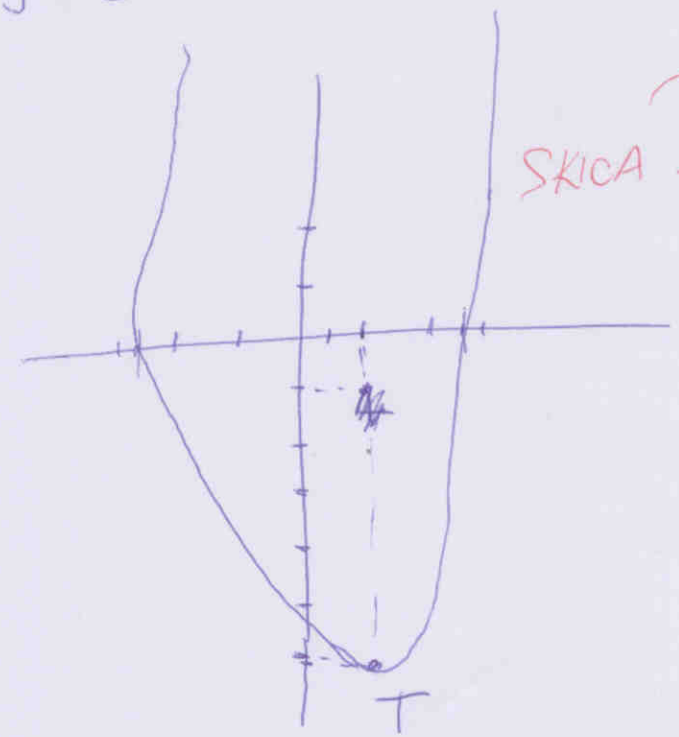
$x_{1,2} = \frac{1 \pm \sqrt{1+28}}{2}$

$x_{1,2} = \frac{1 \pm 5{,}38}{2}$

$x_{1,2} = 2{,}69$

$x_1 = 2{,}69$

$x_2 = -2{,}69$



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5a) $xy' - 4y = x^3$

$x\frac{dy}{dx} - 4y = x^3 \;/:x$

$\frac{dy}{dx} - 4y = \frac{x^3}{x}$

$\frac{dy}{dx} - 4y = x^2 \;/\cdot dx$

$dy - 4y = x^2 dx \;/\int$

$\int dy + 4\int y = \int x^2 dx$

$y + 4\frac{y^2}{2} = \frac{x^3}{3} + c$

$y + 2y^2 = \frac{x^3}{3} + c$

$\int dy - 4y = \int x^2 dx$

$y - 4y = \frac{x^3}{3} + c$

$-3y = \frac{x^3}{3} + c \;/\cdot 3$

$-y = x^3 + c$

$\boxed{y = -x^3 + c}$

2) $\int \frac{x}{(x-1)(x^2+x+1)}\,dx$ $\quad (x+1)^2 = (x^2+x+1)$

$\int \frac{x}{(x-1)(x+1)^2}\,dx = \frac{A}{(x-1)} + \frac{Bx+C}{(x+1)^2}$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{1-4\cdot 1\cdot 1}}{2a}$

$x_{1,2} = \frac{-1 \pm \sqrt{-3}}{2a}$

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(2) $\int \frac{x}{(x-1)(x^2+x+1)}\,dx = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+x+1)} \quad / \cdot (x-1)(x^2+x+1)$

$= A(x^2+x+1) + (Bx+C)(x-1)$

$Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$

$x^2 \equiv A+B = 1 \Rightarrow A = 1B$

$x \equiv A - B + C = 1 \Rightarrow A - B + C = 1$

$1 \equiv A - C = 1$

$A = 1B$

$1 = B$

$\boxed{B = 1}$

$B - B + A = 1$

$\boxed{A = 1}$

$-C = -A$

$C = A$

$\boxed{C = 1}$

$(x^2+x+1) = (x+1)^2$

$\int \frac{x}{(x-1)(x^2+x+1)}\,dx = \frac{1}{(x-1)} + \frac{x+1}{(x^2+x+1)}$

$= \int \frac{1}{(x-1)}\,dx + \int \frac{x+1}{(x+1)^2}\,dx$

$= \int \frac{dx}{(x-1)} + \int \frac{dx}{(x+1)}$

$t = x - 1 \,/\,'$

$dt = dx$

$z = x + 1 \,/\,'$

$dz = dx$

$= \int \frac{dt}{t} + \int \frac{dz}{z}$

$= \ln|t| + \ln|z|$

$= \ln|x-1| + \ln|x+1| + C$

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4 ① NASTAVAK

$$\left[\left(\int \sin 2^2 dx - \int \sin 1^2 dx\right) - \left(\int \sin 2^1 dx - \int \sin 1^1 dx\right)\right]$$

$$\left[\left(4\int \sin dx - \int \sin dx\right) - \left(2\int \sin dx - \int \sin dx\right)\right]$$

$$\left[3\int \sin dx - \int \sin dx\right]$$

$$2\int \sin dx = -2\cos \Rightarrow \boxed{0,999506}$$

4 ② $f(x,y) = x^3 + y^3 - 15xy$

$z_{xx} = 3x^2 - 15y$

$z_{yy} = 3y^2 - 15x$

$z_{yx} =$