MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: STIPE VULIĆ **Broj indeksa:** 57663-2009

Vrijeme: od 9:05 **do** 10:20 ♣3 **Broj bodova:** 0

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. *~~(15)~~* Integriraj

$$\int_1^2 \frac{\sin(\ln x)}{x}dx$$

2. *~~(20)~~* Integriraj

$$\int \frac{x}{(x-1)(x^2+x+1)}dx$$

3. *(20)* Odredi površinu koju zatvaraju krivulja $x = y^2 - 2y + 2$ i pravac $2x + y = 9$.

4. *~~(10+10)~~*

a) Ispitaj ekstreme funkcije

$$f(x, y) = x^3 + y^3 - 15xy$$

b) Odredi domenu funkcije:

$$f(x, y) = \arcsin(x + y)$$

5. *~~(10+15)~~* Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 4y = x^3$$

b)

$$y'' + 9y = 2e^{-3x}$$

VIDI RJEŠENJE 3

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME

Ime i prezime: STIPE VULIĆ Broj indeksa: 57663-2009

4.

b) $f(x,y) = \arcsin(x+y)$

$\arcsin [-1,1]$

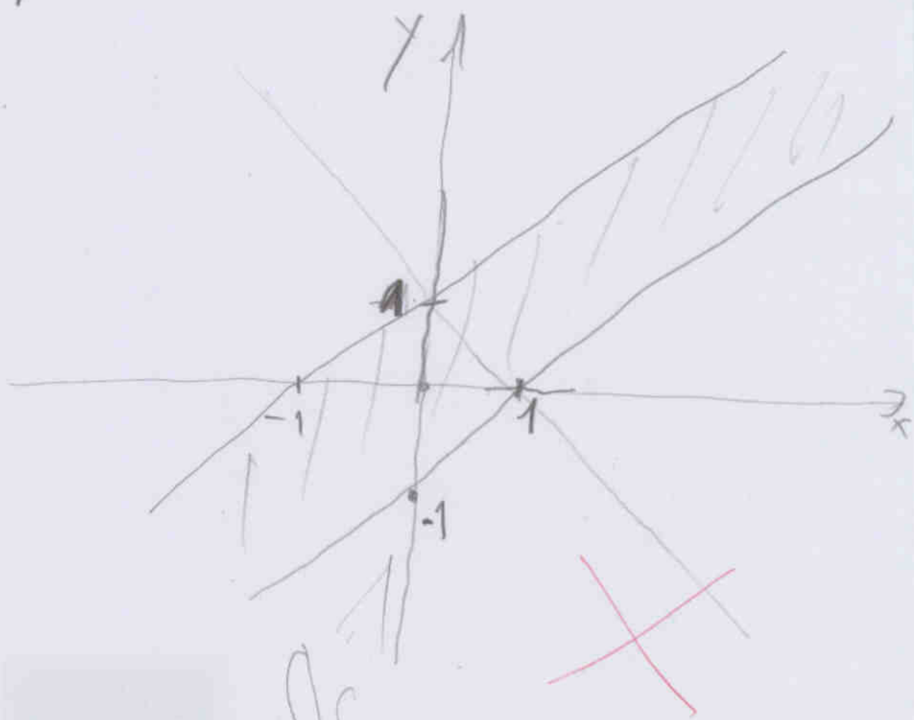
$-1 \geq x+y \geq 1$

$x+y \geq -1$ ×

$x+y \geq 1$

$S(0,0)$

$G=\lambda$



D_f

a) $f(x,y) = x^3 + y^3 - 15xy$

1° PARCIJALNE DERIVACIJE

$f'_x(x,y) = 3x^2 - 15y$

$f'_y(x,y) = 3y^2 - 15x$

2° JEDNAŽBE IZJEDNAČITI S NULOM

$3x^2 - 15y = 0$

$3x^2 = 15y \mid 3$

$x^2 = 5y$

$3y^2 - 15x = 0$

$3y^2 - 15(5y)^2 = 0$

$3y^2 - 75y^2 = 0$

$-72y^2 = 0$

$y^2 = 72$

0

IME I PREZIME: STIPE VULIĆ

BROJ INDEKSA: 57663-2009

$\alpha = 0 \quad \beta = 3$

5. b) $y'' + 9y = 2e^{-3x}$

$y(x) = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$

$\lambda^2 + 9 = 0$

$y(x) = e^{0 \cdot x}(C_1 \cos 3x + C_2 \sin 3x)$

$\lambda^2 = -9$

$y(x) = C_1 \cos 3x + C_2 \sin 3x$

$\lambda = \pm\sqrt{-9}$

$\lambda = \pm 3i$

$y'(x) = C_1(-\sin 3x) \cdot (3x)' + C_2(\cos 3x) \cdot (3x)'$

$y'(x) = C_1(-\sin 3x) \cdot 3 + C_2 \cos 3x \cdot 3$

$y'(x) = -3C_1 \sin 3x (-3) + 3C_2 \cos 3x \cdot 3$

$y''(x) = C_1 \cos 3x + C_2(-\sin 3x)$

$y'' + 9y = 2e^{-3x}$

0

$C_1 \cos 3x + C_2(-\sin 3x) + 9(C_1 \cos 3x + C_2 \sin 3x) = 2e^{-3x}$

$C_1 \cos 3x - C_2(-\sin 3x) + 9C_1 \cos 3x + 9C_2 \sin 3x = 2e^{-3x}$

$\cos 3x (C_1 + 9C_1)$ ~~sin 3x~~ $C_1 + 9C_1$

$\sin 3x (C_2 + 9C_2)$ $C_2 + 9C_2$

0

IME I PREZIME: STIPE VULIĆ BROJ INDEKSA: 57663-2009

3.

$\cancel{x = y^2 - 2y + 2 \qquad 2x + y = 9}$

$\cancel{y^2 - 2y + 2 = 2x + y = 9}$

$y^2 - 2y + 2 = 2x - y - 9 = 0$

5.

a) $xy' - 4y = x^3$

$$y = \left(\frac{C}{\sqrt[4]{x}} \right)'$$

$xy' - 4y = 0$

$xy' = 4y \quad / : x$

$y' = \frac{4y}{x} \quad / \cdot \frac{dx}{y}$

$\frac{dy}{dx} = \frac{1}{4} \frac{y}{x} \quad \Big| \; \frac{dx}{y}$

$\frac{dy}{y} = \frac{1}{4} \frac{dx}{x}$

$\ln|y| = \frac{1}{4} \ln|x| + C$

$\ln|y| = \ln|x^{-4}| + C$

$y = \frac{C}{\sqrt[4]{x}}$

Ime i prezime: STIPE VULIĆ

Broj indeksa: 57663-2009

1.

$$\int_1^2 \frac{\sin \ln(x)}{x}\,dx = \left| \begin{array}{l} \ln x = t \,/' \\ \frac{1}{x}dx = dt \end{array} \right.$$

$$= \boxed{\int_1^2 \sin t \cdot \frac{1}{x}\,dt} \times$$

KOD SUPSTITUCIJE $x \mapsto t$ MORAJU NESTATI SVE POJAVE VARIJABLE x

$$= \frac{1}{x}\int_1^2 \sin t\,dt$$

$\emptyset$

$$= \frac{1}{x}\cdot(-\cos x \ln x) + C$$

$$= -\frac{1}{x}\cdot \cos x \ln x + C \Big|_1^2$$

$$= \left(-\frac{1}{2}\cdot \cos 2 \cdot \ln 2\right) - \left(-\frac{1}{2}\cdot \cos 1 \cdot \ln 1\right)$$

$$= \left(-\frac{1}{2}\cdot 0{,}999 \cdot 0{,}69\right) - \underbrace{\left(-\frac{1}{2}\cdot 0{,}999 \cdot 0\right)}_{0}$$

$$= \left(-\frac{1}{2}\cdot 0{,}68931\right) - ($$

$$= -0{,}5 \cdot 0{,}68931$$

$$= -0{,}344655$$

IME I PREZIME: STIPE VULIĆ BROJ INDEKSA: 57663-2009

2. $\int \frac{x}{(x-1)(x^2+x+1)} dx$

$x = \int \frac{A}{x-1} dx + \int \frac{Bx+C}{x^2+x+1} \quad / \cdot (x-1)(x^2+x+1)$

$x = A(x^2+x+1) + Bx + C(x-1)$

$x = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$

$x = x^2(A+B) + x(A-B+C) + (A-C)$

0

$A+B=0 \Rightarrow B=-A$ $\qquad B=-1$

$A-B+C=1$ $\qquad C=-1$

$A-C=0 \Rightarrow C=-A$

$A-(-A)+C=1$

$A-(-A)+(-A)=1$

$A+A-A=1$

$A=1$

$\int \frac{A}{x+1} dx + \int \frac{Bx+C}{x^2+x+1} dx$

$\int \frac{1}{x+1} dx + \int \frac{(-1)x+(-1)}{x^2+x+1}$

$\int \frac{1}{x+1} dx + \int \frac{-x+(-1)}{x^2+x+1}$

$\Rightarrow$