MATEMATIKA 2

14. lipnja 2012.

Ime i prezime: MARKO VULELIJA **Broj indeksa:** __________

Vrijeme: od 8:50 **do** 10:17 ♣3 **Broj bodova:** 20

Trajanje ispita je 120 minuta. Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata.

1. (15) Integriraj

$$\int_1^2 \frac{\sin(\ln x)}{x}dx$$

2. (20) Integriraj

$$\int \frac{x}{(x-1)(x^2+x+1)}dx$$

3. (20) Odredi površinu koju zatvaraju krivulja $x = y^2 - 2y + 2$ i pravac $2x + y = 9$.

4. (10+10)

a) Ispitaj ekstreme funkcije

$$f(x,y) = x^3 + y^3 - 15xy$$

b) Odredi domenu funkcije:

$$f(x,y) = \arcsin(x+y)$$

5. (10+15) Riješi sljedeće diferencijalne jednadžbe:

a)

$$xy' - 4y = x^3$$

b)

$$y'' + 9y = 2e^{-3x}$$

VIDI RJEŠENJE 3

PISATI JEDNOSTRANO!

NA SVAKI LIST PAPIRA NAPISATI IME I PREZIME

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1. $$\int_1^2 \frac{\sin(\ln x)}{x}\,dx = \qquad \ln x = t \;/\; d \qquad \frac{1}{x}dx = dt$$

$$\int_1^2 \sin t\,dt = \cos t\Big|_1^2 = \cos(\ln x)\Big|_1^2 = 3.77\cdot 10^{-3}$$

Ø

$3.77\cdot 10^{-3}$

$\int \sin t\,dt = -\cos t + C$

POGREŠNO UVRŠTENO

STUPNJEVI UMJESTO RADIJANA

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(2.)

$$\int \frac{x}{(x-1)(x^2+x+1)}dx = \int \frac{A}{x-1} + \int \frac{Bx+C}{(x^2+x+1)}dx$$

$x^2+x+1=t \;/^d$

$(2x+1)dx=dt$

$$\frac{x}{(x-1)(x^2+x+1)} \equiv \frac{A}{x-1} + \frac{Bx+c}{(x^2+x+1)} \quad /(x-1)(x^2+x+1)$$

$x \equiv A(x^2+x+1)+(Bx+c)(x-1)$

$x \equiv Ax^2+Ax+A+Bx^2-Bx+Cx-C$

$x \equiv x^2(A+B)+x(A-B+C)+A-C$

$A+B=0$

$A-B+C=1$

$A-C=0$

$-C=-A$

$C=A$

$\frac{1}{3}-C=0$

$C=\frac{1}{3}$

$A+B=0$

$A-B+A=1$

$A+B=0$

$2A-B=1$

$3A=1$

$A=\frac{1}{3}$

$$I_2 = \int \frac{\frac{2}{3}x+\frac{1}{3}}{(x^2+x+1)}dx$$

$$I_2 = \frac{1}{3}\int \frac{2x+1}{(x^2+x+1)}dx$$

$$I_2 = \frac{1}{3}\int \frac{dt}{t}$$

$I_2 = \frac{1}{3}\ln|t|+c = \frac{1}{3}\ln|x^2+x+1|+c$

$\frac{1}{3}-B+\frac{1}{3}=0$ ✗ 1

$-B+\frac{2}{3}=0$

$B=\frac{2}{3}$

$$\int \frac{\frac{1}{3}}{x-1}dx + \int \frac{\frac{2}{3}x+\frac{1}{3}}{(x^2+x+1)}dx$$

$\frac{1}{3}\int\frac{dx}{x-1} + I_2$

$\frac{1}{3}\ln|x-1|+c+ I_2$

$= \frac{1}{3}\ln|x-1|+c+\frac{1}{3}\ln|x^2+x+1|+c$

$\emptyset$

(5.) (a)

$y''+9y=2e^{-3x}$

$r^2+9r=0$

a b c=0

$$x_{1/2} = \frac{-9\pm\sqrt{81-4\cdot 1\cdot 0}}{2} = \frac{-9\pm\sqrt{81}}{2}$$

$$x_1 = \frac{-9+9}{2} = \frac{0}{2} = \pm\infty$$

$n \neq r_1 \neq r_2$

$y_0 = e(A\cos x + B\sin x)$ ✗

$P(b) = \frac{ke^x}{P(b)}$

$P(b) = a^2+9b$

$= -3^2+9\cdot 3$

$= 9+27$

$= 36$

$P(b) = \frac{2e^{-3x}}{36}$ ✗

$\emptyset$

$y = e^{\alpha x}(A\cos x + B\sin x) + \frac{2e^{-3x}}{36}$

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(3.) $x=y^2-2y+2$

$2x+y=9 \quad = 2x=9-y \quad /:2$

$x=\frac{9}{2}-\frac{y}{2}$

$y^2-2y+2=\frac{9}{2}-\frac{y}{2}$

$y^2-2y+2-\frac{9}{2}+\frac{y}{2}=0$

$y^2-2y+\frac{1}{2}y+2-\frac{9}{2}=0$

$y^2-\frac{3}{2}y-\frac{5}{2}=0$

a b c

$y_{1,2}=\frac{\frac{3}{2}\pm\sqrt{\frac{9}{4}+10}}{2}=\frac{\frac{3}{2}\pm\sqrt{\frac{49}{4}}}{2}$

$y_1=\frac{\frac{3}{2}+\sqrt{\frac{49}{4}}}{2}=\frac{5}{2}$

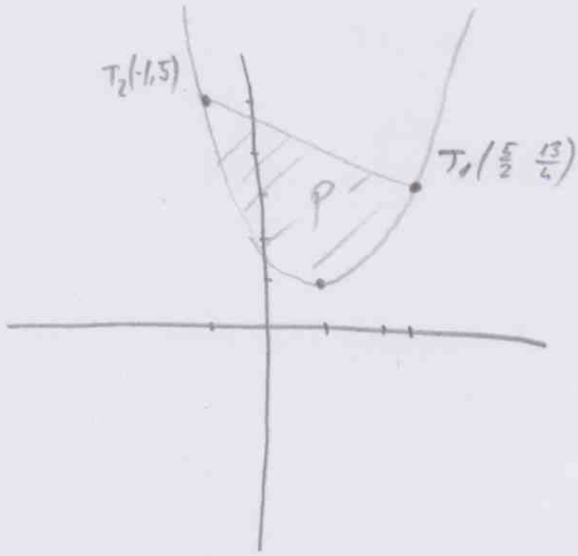
$y_2=\frac{\frac{3}{2}-\sqrt{\frac{49}{4}}}{2}=-1$

$x_1=\frac{9}{2}-\frac{\frac{5}{2}}{2}=\frac{13}{4}$

$x_2=\frac{9}{2}-\frac{-1}{2}=5$

$T_1\left(\frac{5}{2},\frac{13}{4}\right)$

$T_2(-1,5)$



$x=y^2-2y+2$

$x=2y-2$

$2y=2$

$y=1$

$x=1^2-2\cdot1+2$

$x=1-2+2$

$x=1$

20

$P=\int_{-1}^{\frac{5}{2}}\left(\left(\frac{9}{2}-\frac{y}{2}\right)-(y^2-2y+2)\right)dx$

$\frac{-343}{48}$

$P=-\frac{343}{48}$ ✓

$P=\int_{-1}^{\frac{5}{2}}\left(\frac{9}{2}-\frac{y}{2}-y^2+2y-2\right)dx = \int_{-1}^{\frac{5}{2}}\left(-y^2+\frac{3}{2}y+\frac{5}{2}\right)dy = \int_{-1}^{\frac{5}{2}}(-y^2dy)+\frac{3}{2}\int_{-1}^{\frac{5}{2}}ydy+\frac{5}{2}\int dy$

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5. a)

$xy' - 4y = x^3$

$x\frac{dy}{dx} - 4y = x^3 \quad / dx$

$x\,dy - 4y = x^3 dx \quad / \cdot \frac{1}{x}$

$-4y\,dy = \frac{x^3 dx}{x} \quad / \int$

$\int -4y\,dy = \int \frac{x^3 dx}{x}$

$-4\int y\,dy = \int \frac{x^3 dx}{x}$

$-4\frac{y^2}{2} = I_2$

$-2y^2 + C = I_2$

$I_2 = \int \frac{x^3 dx}{x}$

$I_2 = \int x^2 dx$

$I_2 = \frac{x^3}{3} + C$

$-2y^2 + C = \frac{x^3}{3} + C$

4)

$f(x,y) = x^3 + y^3 - 15xy$

$\partial_x f = 3x^2 - 15y$

$\partial_y f = 3y^2 - 15x$

$\partial_{xx} f = 6x$ A

$\partial_{xy} f = -15$ B

$\partial_{yy} f = 6y$ C

$3x^2 - 15y = 0$

$3y^2 - 15x = 0$

$-15y = -3x^2 \quad / (-1)$

$15y = 3x^2 \quad / :15$

$y = \frac{1}{5}x^2$

$3x^2 - 15 \cdot \frac{1}{5}x^2 = 0$

$3x^2 - \frac{15}{5}x^2 = 0$

$3x^2 - 3x^2 = 0$

NEMA EKSTREMA

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