

odgovornosti studenata. **PIŠITE DVOSTRANO!**

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- Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
- Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
- Zadan je trokut s vrhovima $A(-2, 4)$, $B(10, 5)$ i $C(0, -1)$. Izračunati $\oint_{\widehat{ABC}} (x^2 - y) dx + \sin(y^3) dy$. 20
- Izračunati integral funkcije $f(x, y) = \frac{2}{\sqrt{x^2 + y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
- Odrediti integral funkcije $f(x, y) = -y$ na području X koje je ograničeno krivuljama $X \dots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ 20

Tablica integrala

Ukupno:

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{a+x}{a-x} \right + C$
$\int \frac{dx}{x} = \ln x + C$	$\int \sinh x dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln x + \sqrt{x^2 \pm a^2} + C$
$\int \sin x dx = -\cos x + C$	$\int \tanh x dx = \ln \cosh x $	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x dx = \sin x + C$	$\int \coth x dx = \ln \sinh x $	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a} \right) + C$
$\int \tan x dx = -\ln \cos x $	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right]$
$\int \cot x dx = \ln \sin x $	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

20

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s + a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

$$1) f'''(t) + f''(t) = \sin 2t$$

$$f'(0) = 0$$

$$f(0) = f''(0) = 1$$

$$\Delta^3 F(\Delta) - \Delta^2 f(0) - \Delta f'(0) - f''(0) + \Delta^2 F(\Delta) - \Delta f(0) - f'(0) = \frac{2}{\Delta^2 + 4}$$

$$\Delta^3 F(\Delta) - \Delta^2 - 1 + \Delta^2 F(\Delta) - \Delta = \frac{2}{\Delta^2 + 4}$$

$$F(\Delta) (\Delta^3 + \Delta^2) = \frac{2}{\Delta^2 + 4} + \Delta^2 + 1 + \Delta$$

$$F(\Delta) (\Delta^3 + \Delta^2) = \frac{2 + \Delta^4 + \Delta^2 + \Delta^3 + 4\Delta^2 + 4 + 4\Delta}{\Delta^2 + 4}$$

$$F(\Delta) (\Delta^3 + \Delta^2) = \frac{\Delta^4 + \Delta^3 + 5\Delta^2 + 4\Delta + 6}{\Delta^2 + 4} \cdot \left(\frac{1}{\Delta^3 + \Delta^2} \right)$$

$$F(\Delta) = \frac{\Delta^4 + \Delta^3 + 5\Delta^2 + 4\Delta + 6}{\Delta^2(\Delta + 1)(\Delta^2 + 4)} =$$

$$(\Delta + 1)(\Delta^2 + 4)$$

$$\Delta^3 + 4\Delta + \Delta^2 + 4$$

$$\Delta_{1,2} = 0$$

$$\Delta_3 = -1$$

$$\Delta_4 = -4$$

$$\frac{\Delta^4 + \Delta^3 + 5\Delta^2 + 4\Delta + 6}{\Delta^2(\Delta + 1)(\Delta^2 + 4)} = \frac{A}{\Delta} + \frac{B}{\Delta^2} + \frac{C}{\Delta + 1} + \frac{D\Delta + E}{\Delta^2 + 4} \cdot \Delta^2(\Delta + 1)(\Delta^2 + 4)$$

$$\Delta^4 + \Delta^3 + 5\Delta^2 + 4\Delta + 6 = A\Delta(\Delta + 1)(\Delta^2 + 4) + B(\Delta + 1)(\Delta^2 + 4) + C\Delta^2(\Delta^2 + 4) + D\Delta^3(\Delta + 1) + E\Delta^2(\Delta + 1)$$

$$2a \Delta_{1,2} = 0 \quad 6 = 4B \quad \boxed{B = \frac{3}{2}} \quad 2a \Delta_3 = -1 \Rightarrow 7 = 5C \quad \boxed{C = \frac{7}{5}}$$

$$\Delta^4 + \Delta^3 + 5\Delta^2 + 4\Delta + 6 = A\Delta^4 + 4A\Delta^2 + A\Delta^3 + 4A\Delta + B\Delta^3 + 4B\Delta + B\Delta^2 + 4B + C\Delta^4 + 4C\Delta^2 + D\Delta^4 + D\Delta^3 + E\Delta^3 + E\Delta^2$$

$$2a \Delta^4 = 1 = A + C + D \quad D = -A - C + 1 = \frac{1}{2} - \frac{7}{5} + 1 = \frac{5 - 14 + 10}{10} = \frac{1}{10} \quad \boxed{D = \frac{1}{10}}$$

$$2a \Delta^3 = 1 = A + B + D + E$$

$$2a \Delta^2 = 5 = 4A + B + 4C + E \quad E = -A - B - D + 1 = \frac{1}{2} - \frac{3}{2} - \frac{1}{10} + 1 = \frac{5 - 15 - 1 + 10}{10} = -\frac{1}{10} \quad \boxed{E = -\frac{1}{10}}$$

$$2a \Delta = 4 = 4A + 4B = 4A - 4B + 4 = 4A = -\frac{12}{2} + 4 = -2 \quad \boxed{A = -\frac{1}{2}}$$

$$F(\Delta) = -\frac{1}{2} \cdot \frac{1}{\Delta} + \frac{3}{2} \cdot \frac{1}{\Delta^2} + \frac{7}{5} \cdot \frac{1}{\Delta + 1} + \frac{1}{10} \cdot \frac{\Delta}{\Delta^2 + 4} - \frac{1}{10} \cdot \frac{1}{\Delta^2 + 4}$$

$$F(t) = -\frac{1}{2} + \frac{3}{2}t + \frac{7}{5}e^{-t} + \frac{1}{10}\cos(\sqrt{4}t) - \frac{1}{10}\sin(\sqrt{4}t)$$

