MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: TOMISLAV JURIĆ **BROJ INDEKSA:** 57282

Grupa xxoxx POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. ~~20~~
2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. 20
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C} (x^2 - y)\,dx + \sin(y^3)\,dy$. ~~20~~
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. 20
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \dfrac{\pi}{2}x. \end{cases}$ 20

Ukupno: 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\vert\frac{a+x}{a-x}\right\vert + C$
$\int \frac{dx}{x} = \ln\vert x\vert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\vert\frac{x-a}{x+a}\right\vert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\vert x + \sqrt{x^2\pm a^2}\right\vert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\vert\cosh x\vert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\vert\sinh x\vert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\vert\cos x\vert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\vert\sin x\vert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

1) $f'''(t) + f''(t) = \sin 2t$

$f'(0)=0$

$f(0) = f''(0) = 1$

$$s^3F(s) - s^2f(0) - sf'(0) - f''(0) + s^2F(s) - sf(0) - f'(0) = \frac{2}{s^2+4}$$

$$s^3F(s) - s^2 - 1 + s^2F(s) - s = \frac{2}{s^2+4}$$

$$F(s)(s^3+s^2) = \frac{2}{s^2+4} + s^2 + 1 + s$$

$$F(s)(s^3+s^2) = \frac{2+s^4+s^2+s^3+4s^2+4+4s}{s^2+4}$$

$$F(s)(s^3+s^2) = \frac{s^4+s^3+5s^2+4s+6}{s^2+4} \quad \Big/ \cdot \left(\frac{1}{s^3+s^2}\right)$$

$$F(s) = \frac{s^4+s^3+5s^2+4s+6}{s^2(s+1)(s^2+4)} =$$

$(s+1)(s^2+4)$

s^3+4s+s^2+4

$s_{1,2}=0$

$s_3=-1$

$s_4 \neq -4$

$$\frac{s^4+s^3+5s^2+4s+6}{s^2(s+1)(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+4} \quad \Big/ \cdot s^2(s+1)(s^2+4)$$

$$s^4+s^3+5s^2+4s+6 = As(s+1)(s^2+4) + B(s+1)(s^2+4) + Cs^2(s^2+4) + Ds^3(s+1) + Es^2(s+1)$$

za $s_{1,2}=0$ $\quad 6 = 4B \quad \boxed{B=\frac{3}{2}}$ $\quad$ za $s_3=-1 \Rightarrow 7 = 5C \quad \boxed{C=\frac{7}{5}}$

$$s^4+s^3+5s^2+4s+6 = As^4+4As^2+As^3+4As+Bs^3+4Bs+Bs^2+4B+Cs^4+4Cs^2+Ds^4+Ds^3$$
$$+Es^3+Es^2$$

za $s^4 = 1 = A+C+D \qquad D = -A-C+1 = \frac{1}{2} - \frac{7}{5} + 1 = \frac{5-14+10}{10} = \frac{1}{10} \quad \boxed{D=\frac{1}{10}}$

za $s^3 = 1 = A+B+D+E$

za $s^2 = 5 = 4A+B+4C+E \qquad E = -A-B-D+1 = \frac{1}{2} - \frac{3}{2} - \frac{1}{10} + 1 = \frac{5-15-1+10}{10} = \boxed{-\frac{1}{10} E}$

za $s = 4 = 4A+4B \quad = 4A = -4B+4 = 4A = -\frac{12}{2}+4 = -2 \Big/ \cdot \frac{1}{4} = \boxed{A=-\frac{1}{2}}$

$$F(s) = -\frac{1}{2}\cdot\frac{1}{s} + \frac{3}{2}\cdot\frac{1}{s^2} + \frac{7}{5}\cdot\frac{1}{s+1} + \frac{1}{10}\cdot\frac{s}{s^2+4} - \frac{1}{10}\,\frac{1}{s^2+4}$$

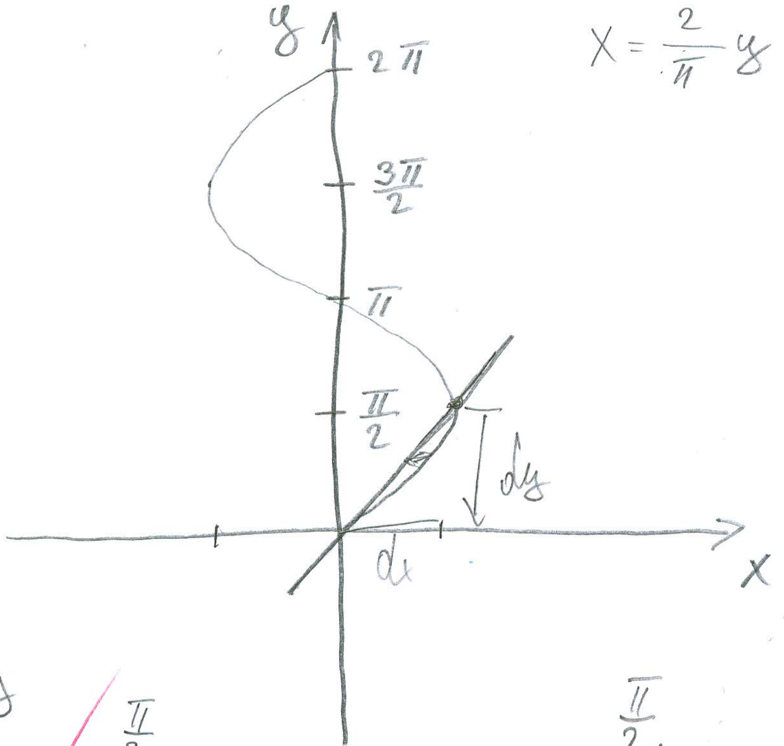
$$F(t) = -\frac{1}{2} + \frac{3}{2}t + \frac{7}{5}e^{-t} + \frac{1}{10}\cos(\sqrt{4}t) - \frac{1}{10}\sin(\sqrt{4}t)$$

5) $f(x,y) = -y$ $\quad X \begin{cases} x = \sin y \\ y = \frac{\pi}{2} x \end{cases}$

$x = 0 \Rightarrow y = \frac{\pi}{2} \cdot 0 = 0$

$x = 1 \Rightarrow y = \frac{\pi}{2} \cdot 1 = \frac{\pi}{2}$

$x = \frac{2}{\pi} y$



$$-\int_0^{\frac{\pi}{2}} y\,dy \int_{\frac{2}{\pi}y}^{\sin y} dx = -\int_0^{\frac{\pi}{2}} y\,dy \left(\sin y - \frac{2}{\pi} y\right) = -\int_0^{\frac{\pi}{2}} \left(y \sin y - \frac{2}{\pi} y^2\right) dy$$

$$\left| \begin{matrix} u = y & dv = \sin y\,dy \\ du = dy & v = -\cos y \end{matrix} \right| = y\cos y - \sin y \Big|_0^{\frac{\pi}{2}} + \frac{2}{3\pi} y^3 \Big|_0^{\frac{\pi}{2}}$$

$\cancel{u \cdot v - \int v\,du}$

$y \cos y - \sin y$

$$= \cancel{\frac{\pi}{2}\cos\frac{\pi}{2}}^{\,0} - \cancel{\sin\frac{\pi}{2}}^{\,1} + \frac{\cancel{2}^{\,1}}{3\cancel{\pi}} \cdot \frac{\pi^{\cancel{3}\,2}}{\cancel{8}_{\,4}}$$

$$-1 + \frac{\pi^2}{12} \quad //$$

20

$$\int_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$

$A(-2,4)$

$B(10,5)$

$C(0,1)$

$\overline{AB}$: $y-4=\frac{4-5}{10+2}(x+2)$

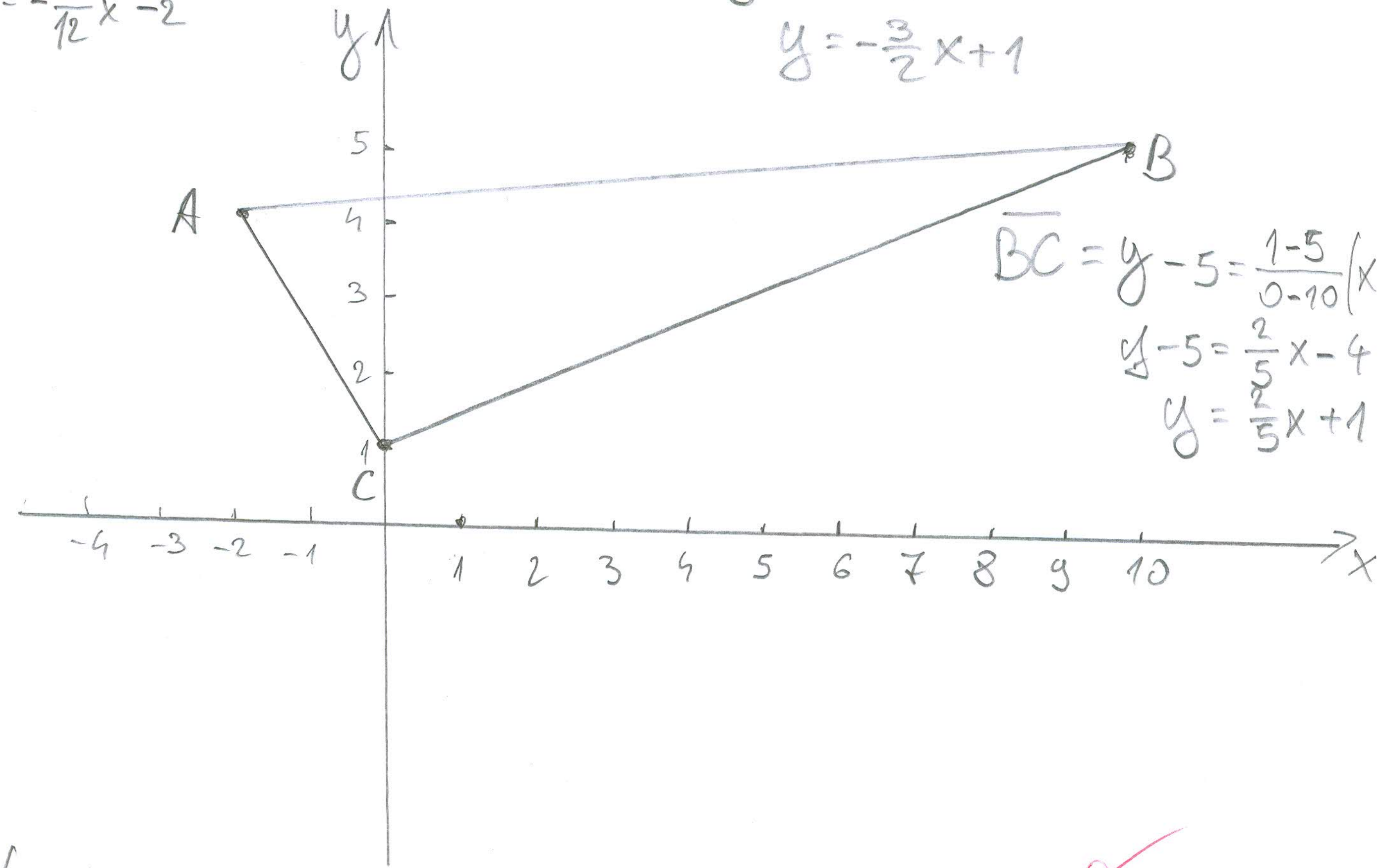
$y-4=-\frac{1}{12}x-6+4$

$y=-\frac{1}{12}x-2$

$\overline{AC} = y-4=\frac{1-4}{0+2}(x+2)$

$y-4=-\frac{3}{2}x-3$

$y=-\frac{3}{2}x+1$



$\overline{BC} = y-5=\frac{1-5}{0-10}(x-10)$

$y-5=\frac{2}{5}x-4$

$y=\frac{2}{5}x+1$

$$\int P\,dx+Q\,dy=\iint\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy$$

$\frac{\partial Q}{\partial x}=3\sin y^2$ $=0$

$\frac{\partial P}{\partial y}=-1$

$$\iint(3\sin y^2+1)\,dx\,dy$$

$$\int_{-2}^{0} 3\sin y\,dx\int_{-\frac{3}{2}x+1}^{-\frac{1}{12}x-2}dy=$$

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POPUNJAVA NASTAVNIK

Broj ↓ bodova

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Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

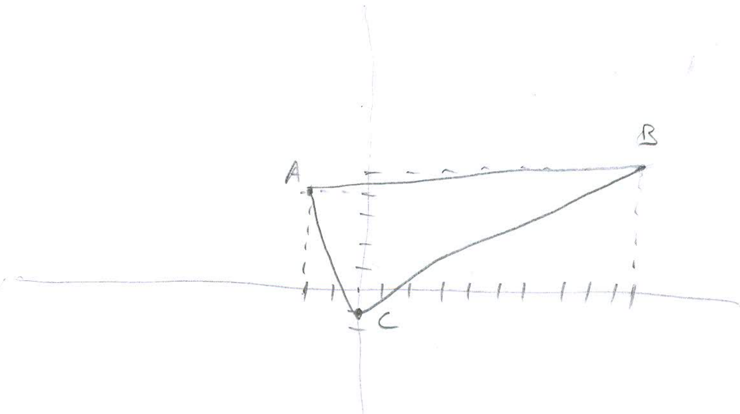
Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

3.

$A(-2,4)$ $B(10,5)$ $C(0,-1)$

$$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$



$\overline{CA}$ $C(\overset{x_1}{0},\overset{y_1}{-1})$, $A(\overset{x_2}{-2},\overset{y_2}{4})$

$$(y-y_1) = \frac{(x_2-x_1)}{(y_2-y_1)}(x-x_1)$$

$$y+1 = \frac{-2-0}{4+1}x$$

$$y+1 = \frac{-2}{5}x$$

$$y = -\frac{2}{5}x - 1$$

$$\frac{2}{5}x = -y-1 \,/\cdot \frac{5}{2}$$

$$x = -\frac{5}{2}y - \frac{5}{2}$$

$\overline{CB}$ $C(\overset{x_1}{0},\overset{y_1}{-1})$, $B(\overset{x_2}{10},\overset{y_2}{5})$

$$y-y_1 = \frac{x_2-x_1}{y_2-y_1}(x-x_1)$$

$$y+1 = \frac{10-0}{5-(-1)}(x-0)$$

$$y+1 = \frac{10}{6}(x-0)$$

$$y+1 = \frac{5}{3}x$$

$$\frac{5}{3}x = y+1 \,/\cdot \frac{3}{5}$$

$$x = \frac{3}{5}y + \frac{3}{5}$$

$(x, y) \longmapsto \left(x, y, \frac{x^2+y^2}{5}\right)$

$(r, \varphi) \longmapsto \left(r\cos\varphi, r\sin\varphi, \frac{r^2}{5}\right)$

2. $x^2 + y^2 = 5z$

$z \leq 1$

$(x, y, z) \mapsto \cancel{(r\cos\varphi, r\sin\varphi)} \Rightarrow \gamma(r, \varphi) = (r\cos\varphi, r\sin\varphi)$

$r \in [0, \sqrt{5}]$

$\varphi \in (0, 2\pi)$

$\frac{\partial\gamma}{\partial r} = (\cos\varphi, \sin\varphi)$ $\times$

$\frac{\partial r}{\partial \varphi} = (-r\sin\varphi, r\cos\varphi)$ $\times$

$\cancel{\emptyset}$

$= \gamma'(r, \varphi) = \begin{bmatrix} \cos\varphi & -r\sin\varphi \\ \sin\varphi & -r\cos\varphi \end{bmatrix} \Rightarrow$

$\Rightarrow \|\gamma'(r, \varphi)\| = \sqrt{r^2+1}$

$2\pi \int_0^{\sqrt{5}} \sqrt{1+r^2}\, dr = \left[\begin{matrix} r = \operatorname{sh} x & 0 \to 0 \\ dr = \operatorname{ch} x\, dx & \sqrt{5} \to \operatorname{arsh}(\sqrt{5}) \end{matrix} \right]$

$= 2\pi \int_0^{\operatorname{arsh}(\sqrt{5})} \underbrace{\sqrt{1+\operatorname{sh}^2 x}}_{\operatorname{ch}^2 x}\, \operatorname{ch} x\, dx = 2\pi \int_0^{\operatorname{arsh}\sqrt{5}} \operatorname{ch}^2 x\, dx =$

$= 2\pi \int_0^{\operatorname{arsh}(\sqrt{5})} \left(\frac{1}{2} + \frac{1}{2}\operatorname{ch}(2x)\right) dx$

$= 2\pi \left(x + \sin h(2x) \Big| \right)$

$= 2\pi \left[\operatorname{arsh}(\sqrt{5}) + \operatorname{sh}(2 \cdot \operatorname{arsh}(\sqrt{5}))\right]$

1.

$$f'''(t) + f''(t) = \sin(2t) \qquad f(0) = 0$$

$$f(0) = f''(0) = 1$$

$$\mathcal{L}(f'''(t)) = s^3 \cdot f(0) - s^2 f(0) - s \cdot f'(0) - f''(0) = 1$$

$$\mathcal{L}(f''(t)) = s^2 \cdot f(0) - s f(0) - f'(0)$$

$$\mathcal{L}(\sin 2t) = \frac{2}{s^2+4}$$

$$s^2 f(s) - s^2 - 1 + s^2 f(s) - s = \frac{2}{s^2+4}$$

$$f(s)(s^2 + s^2) = \frac{2}{s^2+4} + s^2 + s + 1$$

$$\Rightarrow f(s) = \frac{2}{(s^2-4)(s^3+s^2)} + \frac{s^2+s+1}{s^3+s^2}$$

$$\frac{2}{(s^2+4)s^3+s^2} = \frac{2}{s^2(s^2+4)(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{Ds+E}{s^2+4}$$

$$\Rightarrow 2 = s^2(C(s^2+4) + (s+1)(Ds+E)) + A(s^3+s^2+4s+4)s + B(s^3+s^2+4s+4)$$

$$\Rightarrow 2 = 4B$$

$$\left.\begin{aligned} 0 &= 4A + 4B \\ 0 &= 4A + B + 4C + E \\ 0 &= A + B + E + D \\ 0 &= A + C + D \end{aligned}\right\} = 0$$

$A = -\frac{1}{2}$

$B = \frac{1}{2}$

$C = \frac{2}{5}$

$D = \frac{1}{10}$

$E = -\frac{1}{10}$

$$\frac{s-1}{10(s^2+4)} + \frac{1}{s^2-2} - \frac{1}{2s} + \frac{2}{5(s+1)}$$

NASTAVAK ① ZADATKA

$$f(t) = \mathcal{L}^{-1}(f(s)) = \{ \text{IZ TABLICE} \} = (A\ 4)$$

(1) $$\mathcal{L}^{-1}\left[\frac{s-1}{10(s^2+4)}\right] = \mathcal{L}^{-1}\left[-\frac{1}{10(s^2+4)} + \frac{s}{10(s^2+4)}\right] =$$

$$= \frac{5}{10}\mathcal{L}^{-1}\left(\frac{s}{s^2+4}\right) - \frac{1}{10}\mathcal{L}^{-1}\left(\frac{1}{s^2+4}\right) =$$

$$= \frac{1}{10}\cos 2t - \frac{2}{20}\sin 2t$$

(2) $$\mathcal{L}^{-1}\left(\frac{5}{2s^2}\right) = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = \frac{1}{2}t$$

(3) $$\mathcal{L}^{-1}\left(-\frac{1}{2s}\right) = -\frac{1}{2}$$

(4) $$\mathcal{L}^{-1}\left(\frac{2}{5(s+1)}\right) = \frac{2}{5}\mathcal{L}^{-1}\left[\frac{1}{s+1}\right] = \frac{2}{5}e^{-t}$$

(5) $$\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) = e^{-t}$$

(6) $$\mathcal{L}^{-1}\left(\frac{1}{s^2}\right. = t$$

KONAČNO RJEŠENJE

(1) + (2) + (3) + (4) + (5) + (6) = ...

(5)

$f(x,y) = -y$ NA PODRUČJU X KOJE JE OGRANIČENO KRIVULJAMA $X \ldots \begin{cases} x = \sin y \\ y = \frac{\pi}{2} x \end{cases}$

$\Rightarrow \iint_X f(x,y)\,dx\,dy =$

$$\left.\begin{aligned} x &= \sin y \\ y &= \frac{\pi}{2} x \end{aligned}\right\} \Rightarrow x = \sin\left(\frac{\pi}{2}x\right) \Rightarrow \boxed{x = \pm 1}$$

$$\Rightarrow y = \pm \frac{\pi}{2}$$

$$\iint_X f(x,y)\,dy\,dy = \int_{-1}^{0}\left(\int_{0}^{\frac{\pi}{2}} -y\,dy\right)dy =$$

$$\int_{-1}^{0}\left(\int_{-\frac{\pi}{2}}^{0} -y\,dy\right)dx = -\int_{-1}^{0}\left(\frac{y^2}{2}\Bigg|_{-\frac{\pi}{2}}^{0}\right)dx =$$

$$= \frac{\pi^2}{8}\int_{-1}^{0} dy = \frac{\pi^2}{8}$$

$$\int_{0}^{1}\left(\int_{0}^{\frac{\pi}{2}} -y\,dy\right)dy = \{\sin \text{ ch} y\,g\} = \frac{\pi^2}{8}$$

$\Rightarrow$ RJEŠENJE $(1)+(2) = \frac{\pi^2}{4}$

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IME I PREZIME: GREGOR HAMARIĆ **BROJ INDEKSA:** 57650	POPUNJAVA NASTAVNIK Broj ↓ bodova

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2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \leq 1$. ~~20~~
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\overset{\frown}{B}C} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. ~~20~~
4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. ~~20~~
5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \dfrac{\pi}{2}x. \end{cases}$ ~~20~~

Ukupno: 0

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
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$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\lvert\cosh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\lvert\sinh x\rvert$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\lvert\cos x\rvert$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\lvert\sin x\rvert$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
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$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
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$1.\ \varphi'''(t) + \varphi''(t) = \sin 2t$

$\varphi'(0) = 0$

$\varphi''(0) = 1$

$\varphi(0) = 1$

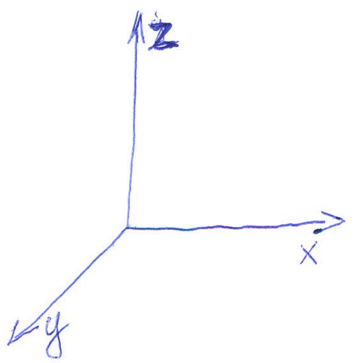
$s^3F(s) - s^2\varphi(0) - s\cancel{\varphi'(0)} - \varphi''(0) + s^2F(s) - s\varphi(0) - \cancel{\varphi'(0)} = \frac{2}{s^2+4}$

$s^3F(s) - s^2 - 1 + s^2F(s) - s = \frac{2}{s^2+4}$

$s^3F(s) - s^2\,1 + F(s) - s - 1 = \frac{2}{s^2+4}$

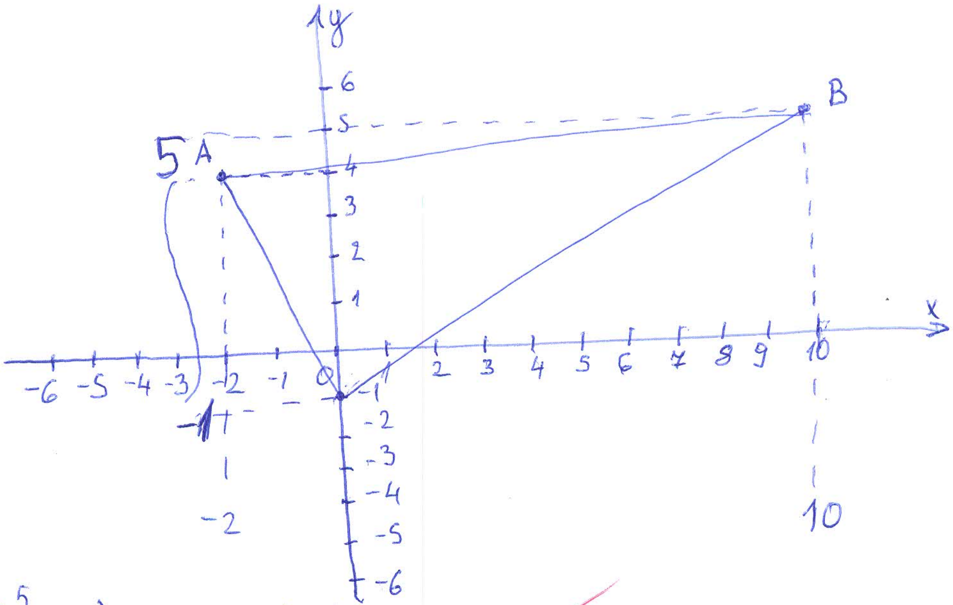
$s\left(s^2F(s) - s + sF(s) - 1 - \frac{1}{s}\right) = \frac{2}{s^2+4}$

$2.\ x^2+y^2=5z\ ,\ z \le 1$



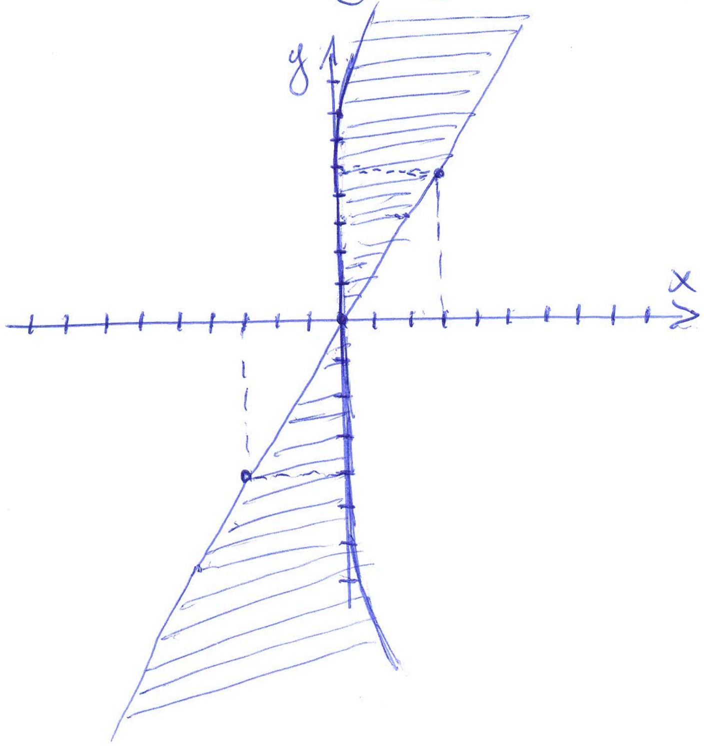
3. A(−2,4) B(10,5) C(0,−1)

$$\oint_{\overset{\frown}{ABC}} (x^2-y)\,dx + \sin(y^3)\,dy$$

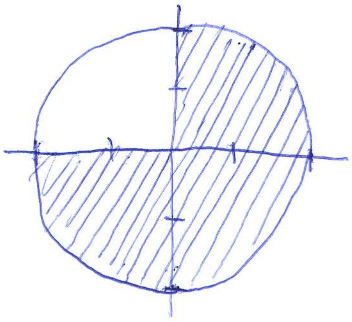


$$\int_{-2}^{10}\int_{-1}^{5} (x^2-y)\,dx + \sin(y^3)\,dy$$

5. $\varphi(x,y) = -y$ x ... $\begin{cases} x = \operatorname{sen} y \\ y = \frac{\pi}{2} x \end{cases}$



4. $f(x,y) = \frac{2}{\sqrt{x^2+y^2}}$; $\varphi \in \left[0, \frac{3\pi}{2}\right]$, $r = 2$



MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: ANĐELA SMOLIĆ **BROJ INDEKSA:** 57283

Grupa xxoxx

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
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Ukupno:

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$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
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1. $f'''(t) + f''(t) = \sin(2t)$

$f'(0) = 0$

$f(0) = f''(0) = 1$

$$s^3F(s) - s^2 \underbrace{f(0)}_{1} - s\underbrace{f'(0)}_{0} - s\underbrace{f''(0)}_{1} + s^2F(s) - s\underbrace{f(0)}_{1} - \underbrace{f'(0)}_{0} = \frac{2}{s^2+4}$$

$$s^3F(s) - s^2F_s = \frac{2}{s^2+4}$$

$$F(s)(s^3 - s^2) = \frac{2}{s^2+4}$$

$$\frac{2}{(s-2)(s+2)s(s^2-5)}$$

$$F(s) = \frac{\frac{2}{s^2+4}}{\frac{s^3-s^2}{1}} = \frac{2}{(s^2+4)(s^3-s^2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2} + \frac{D_{E+F}}{s^2+5}$$

$$2 = A s^2(s-2)(s^2+5) + B s(s-2)(s^2+5) + C s s^2 (s^2+5) + (D_{E+F}) s s^2 (s-2)$$

$$2 = A(s^3 - 2s^2)(s^2+5) + B(s^2-2s)(s^2+5) + Cs^3(s^2+5) + (D_{E+F}) s^3(s-2)$$

$$2 = A(s^5 + 5s^3 - 2s^4 - 10s^2) + B(s^4 + 5s^2 - 2s^3 - 10s) + Cs^5 + s^4 + (D_{E+F}) s^4 - 2s^3$$

$$2 = As^5 + 5As^3 - 2As^4 - 10As^2 + Bs^4 + 5Bs^2 - 2Bs^3 - 10Bs +$$
$$Cs^5 + Cs^4 + D_{E+F}s^4 - 2D_{E+F}s^3$$

$$2 = s^5(A+C) + s^4(2A+B+C+(D_{E+F})) + s^3(5A-2B-2(D_{E+F}))$$
$$+ s^2(-10A+5B) - s(10B)$$

$A + C = 0$

$2A + B + C + D_{E+F} = 0$

$5A - 2B - 2D_{E+F} = 0$

$-10A + 5B = 0$

$-10B = 2 \;/ : (-10)$

$B = -\frac{2}{10} = -\frac{1}{5}$

$$\boxed{B = -\frac{1}{5}}$$

$-10A + 5B = 0$

$-10A + 5\cdot\left(-\frac{1}{5}\right) = 0$

$-10A - 1 = 0$

$-10A = 1 \quad /:(-10)$

$\boxed{A = -\frac{1}{10}}$

$A + C = 0$

$-\frac{1}{10} + C = 0$

$-\frac{1}{10} = -C \quad /:(-)$

$\boxed{C = \frac{1}{10}}$

$2A + B + C + D_{E+F} = 0$

$2\cdot\left(-\frac{1}{10}\right) - \frac{1}{5} + \frac{1}{10} + D_{E+F} = 0$

$-\frac{2}{10} - \frac{1}{5} + \frac{1}{10} + D_{E+F} = 0$

$\frac{-2-2+1}{10} + D_{E+F} = 0$

$\frac{-3}{10} + D_{E+F} = 0$

$\boxed{D_{(E+F)} = \frac{3}{10}}$

$f(s) = \frac{-\frac{1}{10}}{s} + \frac{-\frac{1}{5}}{s^2} + \frac{\frac{1}{10}}{s-2} + \frac{\frac{3}{10}}{s^2+s}$

$f(s) = -\frac{1}{10}\cdot\frac{1}{s} + \left(-\frac{1}{5}\right)\cdot\frac{1}{s^2} + \frac{1}{10}\cdot\frac{1}{s-2} + \frac{3}{10}\cdot\frac{1}{s^2+s}$

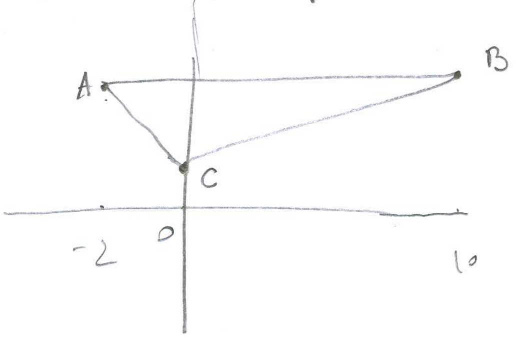
$f(s) = -\frac{1}{10}\cdot 1 + \left(-\frac{1}{5}\right)\cdot t + \frac{1}{10}\cdot e^{at} + \frac{3}{10}\cdot \sinh(at)$

$f(s) = -\frac{1}{10} - \frac{1}{5}t + \frac{1}{10}e^{at} + \frac{3}{10}\sinh(at) = \sin(2t)$ //

X

$A(-2,4) \quad B(10,5) \quad C(0,-1)$

$$\oint_{ABC} (x^2-y)\,dx + \sin(y^3)\,dy$$



AB
AC
BC

$A(\overset{x_1}{-2},\overset{y_1}{4}) \quad B(\overset{x_2}{10},\overset{y_2}{5})$

AB... $y-y_1=\dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$

$$y-4=\frac{5-4}{10+2}(x+2)$$

$$y-4=\frac{1}{12}(x+2)$$

$$y-4=\frac{1}{12}x+\frac{1}{6}$$

$$y=\frac{1}{12}x+\frac{1}{6}+4$$

AB... $y=\dfrac{1}{12}x+\dfrac{25}{6}$

AC $\quad A(\overset{x_1}{-2},\overset{y_1}{4}) \quad C(0,-1)$

AC... $y-4=\dfrac{-1-4}{0+2}(x+2)$

$$y-4=\frac{-5}{2}(x+2)$$

$$y-4=-\frac{5}{2}x-5$$

AC... $y=-\dfrac{5}{2}x-1$

BC... $B(\overset{x_1}{10},\overset{y_1}{5}) \quad C(0,-1)$

$$y-5=\frac{-1-5}{0-10}(x-10)$$

$$y-5=\frac{-6}{-10}x-6$$

BC... $y=\dfrac{3}{5}x-1$

$$\int_{-2}^{0}\int_{-\frac{5}{2}x-1}^{\frac{1}{12}x+\frac{25}{6}} \quad \int_{0}^{10}\int_{\frac{3}{5}x-1}^{\frac{1}{12}x+\frac{25}{6}} \quad \cancel{(x^2-y)\,dx+\sin(y^3)\,dy}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: BRUNO LIPOTICA **BROJ INDEKSA:** 54960	POPUNJAVA NASTAVNIK Broj ↓ bodova

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$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

(4) $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$, $\left(\varphi \in \left[0, \frac{3\pi}{2}\right]\right)$

$x^2 = \sin\varphi$
$y^2 = \cos\varphi$

$$f(x,y) = \frac{2}{\sqrt{(\sin^2\varphi + \cos^2\varphi)}}$$

$$= \frac{2}{\sqrt{1}} = 2$$

$$\int_0^{3\frac{\pi}{2}} 2 \, dy = 2 \int_0^{\frac{3\pi}{2}} = 2 \cdot \Big|_0^{\frac{3\pi}{2}}$$

$$= \underline{\underline{3\pi}}$$

$+y''(t) = \sin(2t)$

$(0) = f''(0) = 1$

$-s^2 \overset{1}{f(0)} - s \overset{0}{f'(0)} - \overset{1}{f''(0)} + s^2 F(s) -$

$\overset{1}{f(0)} - \overset{0}{f'(0)} = \frac{2}{s^2+4}$

$$s^3 F(s) - s^2 - 1 + s^2 F(s) - s = \frac{2}{s^2+4}$$

$$s^3 F(s) + s^2 F(s) = s^2 + s + 1 + \frac{2}{s^2+4}$$

$$F(s)(s^2+s) = s^2 + s + 1 + \frac{2}{s^2+4} \quad \Big| \cdot (s^2+4)$$

$$F(s)(s^2+s) = s^2(s^2+4) + s(s^2+4) + 1(s^2+4) + 2$$

$$F(s)(s^2+s) = s^4 + 4s^2 + s^3 + 4s + s^2 + 4 + 2$$

$$F_s(s^2+s) = \frac{A}{s} + \frac{B}{s} + \frac{C}{s}$$

$$= A(s^3) + B$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: Riko Kolega BROJ INDEKSA: ~~55849-2008~~ 0269/788	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. **20**

2. Izračunajte površinu oplošja paraboloida $x^2 + y^2 = 5z$, $z \le 1$. **20**

3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{\widehat{ABC}} (x^2 - y)\,\mathrm{d}x + \sin(y^3)\,\mathrm{d}y$. **20**

4. Izračunati integral funkcije $f(x,y) = \dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi \in [0, \frac{3\pi}{2}]$) radijusa $r = 2$ sa središtem u ishodištu. **20**

5. Odrediti integral funkcije $f(x,y) = -y$ na području X koje je ograničeno krivuljama $X \ldots \begin{cases} x = \sin y, \\ y = \frac{\pi}{2}x. \end{cases}$ **20**

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\|x + \sqrt{x^2 \pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**	Grupa xxoxx
IME I PREZIME: MAGDA MANDIĆ **BROJ INDEKSA:** 0269015993 / 55690 - 2008.	POPUNJAVA NASTAVNIK Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t)+f''(t)=\sin(2t)$, $f'(0)=0$ i $f(0)=f''(0)=1$. — 20
2. Izračunajte površinu oplošja paraboloida $x^2+y^2=5z$, $z\le 1$. — 20
3. Zadan je trokut s vrhovima $A(-2,4)$ $B(10,5)$ i $C(0,-1)$. Izracunati $\oint\limits_{A\widehat{B}C}(x^2-y)\,\mathrm{d}x+\sin(y^3)\,\mathrm{d}y$. — 20
4. Izračunati integral funkcije $f(x,y)=\dfrac{2}{\sqrt{x^2+y^2}}$ na prve tri četvrtine kruga ($\varphi\in[0,\frac{3\pi}{2}]$) radijusa $r=2$ sa središtem u ishodištu. — 20
5. Odrediti integral funkcije $f(x,y)=-y$ na području X koje je ograničeno krivuljama $X\ldots\begin{cases}x=\sin y,\\ y=\dfrac{\pi}{2}x.\end{cases}$ — 20

Ukupno:

Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\ne -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
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$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
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t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a}F(\frac{s}{a})$
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e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Mateja Mitrović BROJ INDEKSA: 0269037541

Grupa xxoxx

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: $f'''(t) + f''(t) = \sin(2t)$, $f'(0) = 0$ i $f(0) = f''(0) = 1$. 20
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Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2\ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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