MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: STIPE PERKOVIĆ **BROJ INDEKSA:** 56510

Grupa xxoox — POPUNJAVA NASTAVNIK — Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $z = y$ i $z = x - 2$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\ x'(0) = 0.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \le 1, -1 \le z \le 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} 2x \, dy dz$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ ~~20~~

Ukupno: 55

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln \lvert \cosh x \rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln \lvert \sinh x \rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \lvert \sin x \rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a\,t)$	$\frac{1}{a} F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) \, dq$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{F(s)}{s}$
$(1-at)\,e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s) - s^2f(0) - sf'(0) - f''(0)$

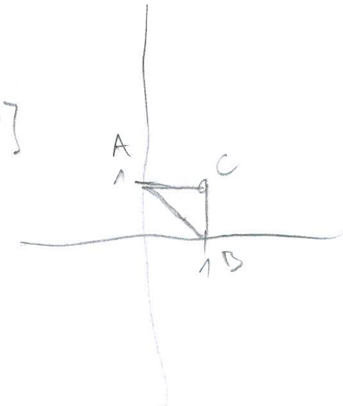
1.

$$\iint_S e^{x+y}\,dx\,dy$$

$A(0,1)$ $B(1,0)$ $C(1,1)$

$x\in[0,1]$

$y\in[1-x,1]$



$$\int_0^1\int_{1-x}^1 e^{x+y}\,dy\,dx=\int_0^1 e^x\cdot(e^1-e^{1-x})\,dx$$

15

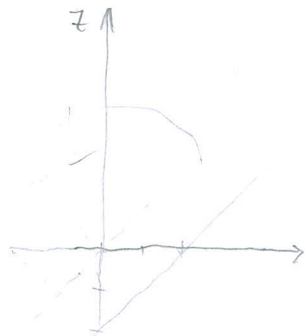
2.

$x^2+y^2=4$; $z=y$, $z=x-2$

$r\in[0,2]$

$\varphi\in[0,2\pi]$

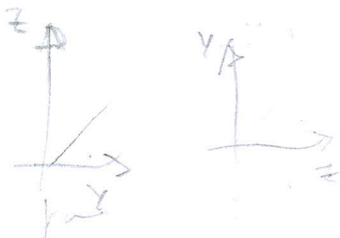
$z\in[r\cos\varphi-2,\ r\cos\varphi]$



$$\int_0^{2\pi}\int_0^2 r\int_{r\cos\varphi-2}^{r\cos\varphi} 1\,dz\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^2 r^2\cos\varphi-r^2\cos\varphi+2r\,dr\,d\varphi=\int_0^{2\pi}\left(\frac{8}{3}\cos\varphi-\frac{8}{3}\cos\varphi+4\right)d\varphi=\frac{8}{3}\sin(2\pi)-\frac{2}{3}\sin(2\pi)+8\pi$$

$=8\pi=25{,}13$



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(5) $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$

$\frac{\partial f}{\partial x} = \frac{\sin y}{x} \quad / \int dx$

$\underline{f(x,y) = \sin y \ln x + C(y)}$

$\frac{\partial f}{\partial y} = \ln x \cos y$

$C'(y) = \cos y \quad / \int dy$

$\underline{f(x,y) = \sin y \ln x + \sin y}$

$f(1,0) - f(e,\pi)$

$\underbrace{\sin(0)\ln|1|}_{0} + \underbrace{\sin(0)}_{0} - (\underbrace{\sin(\pi)\ln|e|}_{0} + \underbrace{\sin(\pi)}_{0})$

$= \underline{\underline{0}}$

(4.) $x'''(t) + x'(t) = 0 \quad ; \; x(0) = x''(0) = 1 \quad ; \; x'(0) = 0$

$s^3 X(s) - s^2 \underbrace{x(0)}_{1} - s\underbrace{x'(0)}_{0} - \underbrace{x''(0)}_{1} + sX(s) - \underbrace{x(0)}_{1} = 0$

$s^3 X(s) - s^2 - 1 + sX(s) - 1 = 0$

$X(s) = \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / s(s^2+1)$

$s^2 + 2 = A(s^2+1) + (Bs+C)s$

$s^2 + 2 = As^2 + A + Bs^2 + Cs$

$2 = A$

$0 = C$

$1 = A + B \Rightarrow B = -1$

$X(s) = \frac{2}{s} + \frac{-s}{s^2+1}$

$\underline{\underline{x(t) = 2 - \cos(t)}}$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: Roko Burčul **BROJ INDEKSA:** 55820-2008

Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

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Ukupno: 35

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③ $x'''(t) + x'(t) = 0$, $x(0) = x''(0) = 1$, $x'(0) = 0$

$x'''(t) = s^3 X(s) - s^2 x(0) - s\,x'(0) - x''(0)$

$x'''(t) = s^3 X(s) - s^2 \cdot 1 - s \cdot 0 - 1$

$x'''(t) = s^3 X(s) - s^2 - 1$

$x'(t) = s X(s) - x(0)$

$x'(t) = s X(s) - 1$

$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$

$s^3 + s = s(s^2+1)$

$s^3 X(s) + s X(s) = s^2 + 2$

$X(s)(s^3+s) = s^2+2 \quad / : (s^3+s)$

$$X(s) = \frac{s^2+2}{s^3+s}$$

$$X(s) = \frac{s^2+2}{s(s^2+1)}$$

$$\frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$$

$s^2+2 = As^2 + A + Bs^2 + Cs$

$\frac{s}{s^2+a^2} = \cos(at)$

1^2

$A = 2$

$C = 0$

$A + B = 1$

$B = 1 - A$

$B = 1 - 2$

$B = -1$

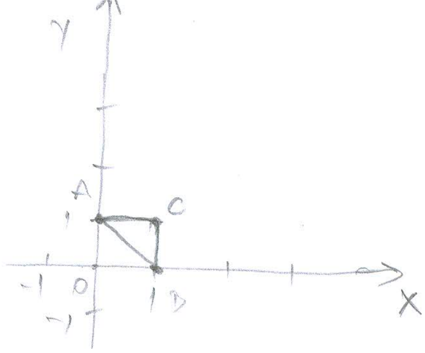
$$\frac{A}{s} + \frac{Bs+C}{s^2+1} = 2 \cdot \frac{1}{s} - 1 \cdot \frac{s}{s^2+1} + 0$$

$$= 2 \cdot 1 - 1 \cdot \cos(t) = 2 - \cos(t)$$

① $\iint\limits_S e^{x+y}\,dx\,dy$

S

A(0, 1)
B(1, 0)
C(1, 1)



A(0,1)
B(1,0)

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1=\frac{0-1}{1-0}(x-0)$

$y-1=-x$

$y=-x+1$

B(1,0)
C(1,1)

$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-0=\frac{1-0}{1-1}(x-1)$

$y=0$

A(0,1)
C(1,1)

$y-1=\frac{1-1}{1-0}(x-0)$

$y-1=0$

$y=1$

$$\int_0^1\int_{-x+1}^{1} e^{x+y}\,dy\,dx=\int_0^1\int_{-x+1}^{1} e^{x}\cdot e^{y}\,dy\,dx=\int_0^1 e^{x}\cdot e^{y}\Big|_{-x+1}^{1}\,dx=$$

15

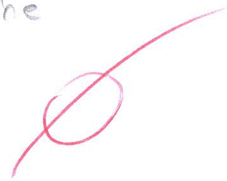
$$=\int_0^1$$

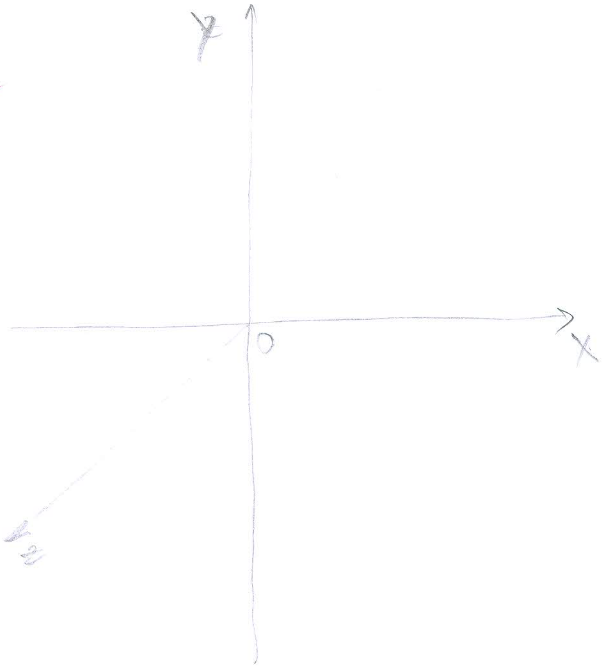
DALJE...

(2) $x^2+y^2=4$ valjak

$z=y$, $z=x-2$ ravnine

$x^2+y^2=R^2$





MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!** **IME I PREZIME:** NIKOLA MILUTIN **BROJ INDEKSA:** 58150	Grupa xxoox POPUNJAVA NASTAVNIK Broj ↓ bodova

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Ukupno: 35

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$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

(3) $x'''(t) + x'(t) = 0$

$x(0) = 1$

$x''(0) = 1$

$x'(0) = 0$

$$s^3 X(s) - s^2 \underbrace{x(0)}_{1} - \cancel{s x'(0)} - \underbrace{x''(0)}_{1} + s X(s) - \underbrace{x(0)}_{1} = 0$$

$$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$$

$$X(s)\left(\underbrace{s^3 + s}_{s(s^2+1)}\right) = s^2 + 2 \quad /:(s(s^2+1))$$

$$X(s) = \frac{s^2+2}{s(s^2+1)}$$

$$\frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad /\cdot(s(s^2+1))$$

$$s^2 + 2 = A(s^2+1) + Bs^2 + Cs$$

* NASTAVAK LAPLASOVE NA DRUGOJ STRANI !!!

(1) $\iint_S e^{x+y}\,dx\,dy$ $S =$ A $(0, 1)$, B $(1, 0)$, C $(1, 1)$

$$\iint_S e^x \cdot e^y \,dx\,dy$$

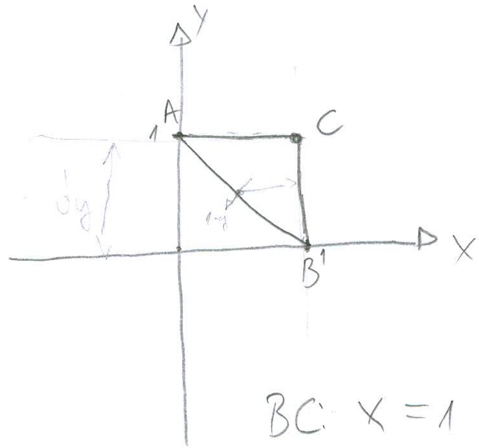
$$= \int_0^1 e^y\,dy \cdot \int_{1-y}^1 e^x\,dx$$

$$= \int_0^1 e^y\,dy \cdot \int (e^1 - e^{1-y})\,dx$$

$$= \int (e^1 - e^0)\,dy\,(e^1 - e^{1-y})\,dx$$

$$= \int (e-1)(e$$

$\emptyset$



BC: $x = 1$

AC: $y = 1$

AB: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{0-1}{1-0}(x - 0)$

$y - 1 = -x$

$\boxed{x = 1 - y}$

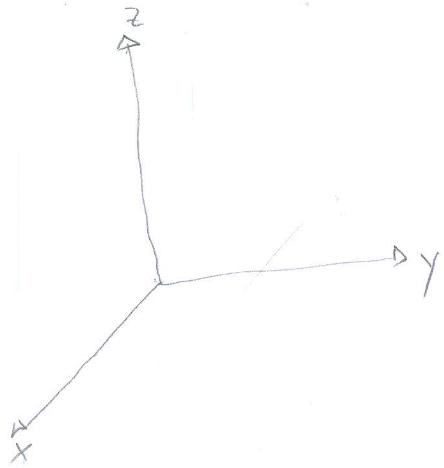
(2.) VALJAK $x^2+y^2=4$ i ravnine $z=y$ i $z=x-2$

$z = y = r\sin\varphi$, $\quad z = x-2 = r\cos\varphi - 2$

$r^2=4$

$r=2$

$x = r\cos\varphi$
$y = r\sin\varphi$
$z = z$



GORNJA - DONJA

15

$$V=\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_{r\cos\varphi-2}^{r\sin\varphi} dz$$

$$V=\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int (r\sin\varphi - r\cos\varphi+2)\,dz$$

$$V=\int_0^{2\pi} d\varphi \int_0^2 r\left(\frac{r^2}{2}\sin\varphi - \frac{r^2}{2}\cos\varphi+2r\right)dr$$

$r\in[0,2]$
$\varphi\in[0,2\pi]$
$z\in[r\cos\varphi-2,\ r\sin\varphi]$

$$V=\int_0^{2\pi} d\varphi \int_0^2 \left(\frac{r^3}{2}\sin\varphi - \frac{r^3}{2}\cos\varphi+2r^2\right)dr$$

$$V=\int_0^{2\pi} d\varphi \left(\frac{r^3}{2}\sin\varphi - \frac{r^3}{2}\cos\varphi+2r^2\right)\Big|_0^2$$

$$V=\int_0^{2\pi} d\varphi\,(4\sin\varphi - 4\cos\varphi+8) \Rightarrow \int_0^{2\pi} d\varphi\; 4(\sin\varphi-\cos\varphi+2)$$

(3.) NASTAVAK LAPLACEOVE

$s^2+2 = As^2+A+Bs^2+Cs$

$1=A+B \quad -B=A-1 /\cdot(-1) \Rightarrow B=-A+1$ $\quad \boxed{B=-1}$

$0=C \Rightarrow \boxed{C=0}$

$2=A \Rightarrow \boxed{A=2}$

$$X(s)=\left\{2\cdot\frac{1}{s}+(-1)\,\frac{s}{s^2+1}\right\}$$

$$X(t)=L^{-1}\left\{2\cdot\frac{1}{s}-\frac{s}{s^2+1}\right\}$$

$$x(t)=2-\cos(t)$$

$$\Rightarrow 4\int_0^{2\pi}(\sin\varphi-\cos\varphi+2)\,d\varphi$$

$$\Rightarrow 4\,(\sin\varphi-\cos\varphi+2\varphi)\Big|_0^{2\pi}$$

$$\Rightarrow 4\cdot(\underbrace{\sin 2\pi}_{0}-\underbrace{\cos 2\pi}_{0}+4\pi)-(\underbrace{\sin 0}_{0}-\underbrace{\cos 0}_{1})$$

$$\Rightarrow 4\cdot((4\pi)+(1))$$

$$\Rightarrow 8\pi+4 \approx 29{,}133$$

⑤ (e,π)

$$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y \, dy$$

$$W\begin{bmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{bmatrix} - \operatorname{grad} f$$

$$\partial x f = -\frac{\sin y}{x} \Big/ \int dx$$

$$f = -\ln|x| + C(y)$$

DONJA - GORNJA

$$f\partial y = -\ln x \cos y$$

$$\frac{\partial}{\partial y}\left(-\ln|x| + C(y)\right) = -\ln x \cos y$$

$$\frac{\partial C(y)}{\partial y} = -\ln x \cos y \Big/ \cdot \left(\frac{\partial}{\partial y}\right)$$

$$C(y) = \sin y$$

$$f = -\ln|x| + \sin y$$

$$f(1,0) - f(e,\pi) = f(-\ln|x|) - f(-\ln|x| + \sin e) - (-\ln|x|)$$

$$= -\ln|x| - (-\cancel{\ln|x|} + \cancel{\ln x})$$

$$= \underline{-\ln|x|}$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: LUKA MARĐETKO

BROJ INDEKSA: 55821-2008

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20

2. Izracunati volumen tijela omeđenog valjkom $x^2 + y^2 = 4$ i ravninama $z = y$ i $z = x - 2$. 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20

$$x'''(t) + x'(t) = 0, \quad x(0) = x''(0) = 1,\, x'(0) = 0.$$

4. Neka je C cilindar zadan sa $C = \{(x,y,z) : (x+2)^2 + (y-3)^2 \leq 1, -1 \leq z \leq 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\partial C} 2x \, dy dz$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ 20

Ukupno: 20

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, \; n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos\left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \, dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) \, dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

③ $x'''(t) + x'(t) = 0$

$x'(0) = 0$

$= s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$

$= s^3 F(s) - s^2 f(0) - 0 - 1$ ⊛

$= sF(s) - f(0)$

$= sF(s) - 1$ ⊛

$\Rightarrow s^3 F(s) - s^2 f(0) - 0 - 1 + sF(s) - 1$

$\Rightarrow s^3 F(s) - s^2 f(0) + sF(s) - 2$

$\Rightarrow s^3 F(s) + sF(s) = s^2 f(0) + 2$

$= \frac{s^2+2}{s^3+s} = \frac{s^2+2}{s(s^2+1)}$

$= \frac{s^2+2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} \quad / \cdot s(s^2+1)$

$\Rightarrow s^2+2 = A(s^2+1) + Bs+C \cdot s$

$\Rightarrow s^2+2 = As^2 + A + Bs^2 + C \cdot s$

$s^2+2 = (A+B) \cdot s^2 + Cs + A$

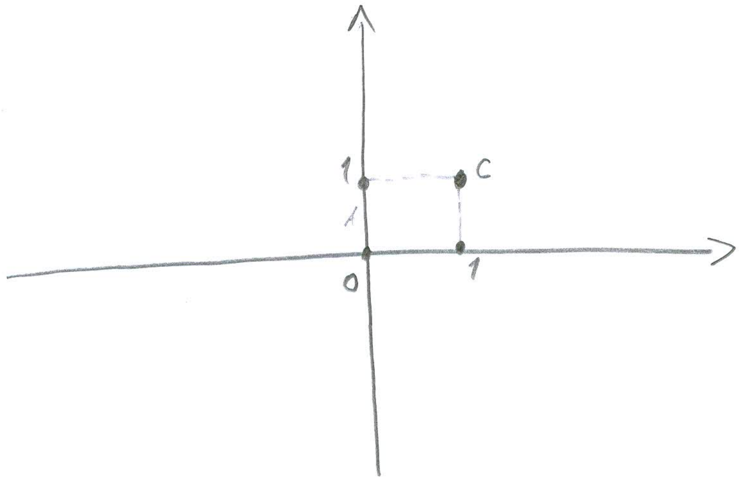
$A+B=1 \quad C=0 \quad A=2 \qquad 2+B=1$

$B = 1-2 = -1$

$\Rightarrow \frac{2}{s} - \frac{1 \cdot s}{s^2+2} = 2 \cdot \frac{1}{s} - \frac{s}{s^2+2}$

$= 2 - \cos(t)$ //

1. $\iint\limits_S e^{x+y}\,dx\,dy \qquad A(0,1) \quad B(1,0) \quad C(1,1)$



$A(0,1)$
$B(1,0)$

$$y-1=\frac{0-1}{1-0}(x-0)$$

$$y-1=\frac{-1}{1}(x-0)$$

$$y-1=-x$$

$$\underline{y=-x+1}$$

$A(0,1)$
$C(1,1)$

$$y-1=\frac{1-1}{1-0}(x-0)$$

$$y-1=\cancel{\frac{0}{1}(x-0)}$$

$$\underline{y=1}$$

$B(1,0)$
$C(1,1)$

$$y-0=\cancel{\frac{1-1}{1-0}(x-1)}$$

$$\underline{y=0}$$

$$=\iint\limits_S e^{x+y}\,dx\,dy=\int_0^1\cdot\int_{-x+1}^{1}e^{x+y}\,dx\,dy$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: MARINO PRENDŽA

BROJ INDEKSA: 57659

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~

2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. ~~20~~

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: ~~20~~

$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=1,\ x'(0)=0.$$

4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2 \le 1,\ -1 \le z \le 1\}$. Izračunati plošni integral 20

$$\iint\limits_{\widehat{\partial C}} 2x \, dydz$$

5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ ~~20~~ 19

Ukupno: 19

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln \lvert x \rvert + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
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$\int \tan x \, dx = -\ln \lvert \cos x \rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
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t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
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$(1-at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

3. $x'''(t) + x'(t) = 0 \qquad x(0) = 1 \quad x''(0) = 1 \quad x'(0) = 0$

$s^3X(s) - s^2x(0) - s\cancel{x'(0)}^{0} - x''(0) + \underbrace{(s^2X(s) - s\,x(0) - \cancel{x'(0)}^{0})}_{\times} = 0$

$s^3X(s) - s^2 - 1 + s^2X(s) - s = 0$

$X(s)(s^3 + s^2) = s^2 + 1 + s \;/ : (s^3 + s^2)$

$$X(s) = \frac{s^2+s+1}{s^2(s+1)}$$

$\frac{s^2+s+1}{s^2(s+1)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{s+1}$

$s^2+s+1 = A(s+1) + Bs(s+1) + Cs^2$

$s^2+s+1 = As + A + Bs^2 + Bs + Cs^2$

$1 = B + C \qquad\qquad B + C = 1$

$1 = A + B \qquad A + B = 1 \qquad \boxed{C = 1}$

$1 = A \rightarrow \boxed{A = 1} \quad 1 + B = 1$

$B = 1 - 1$

$B = 0$

$X(s) = \frac{1}{s^2} + \frac{0}{s} + \frac{1}{s+1}$

$X(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2} + \frac{1}{s+1}\right\}$

$X(t) = t + e^{-t}$ ✗

0

5. $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln\cancel{x}\cos y\,dy$

$W = \begin{bmatrix} \frac{\sin y}{x} \\ \ln\cos y \end{bmatrix} = -\operatorname{grad} f$

$d_x f = -\frac{\sin y}{x} \;/ \int dx$

$f = -\sin y \ln x + C(y)$ ✓

$d_y f = -\ln\cos y$

$\frac{\partial}{\partial(y)}(-\sin y \ln x + C(y)) = -\ln\cos y$

$-\cancel{\ln\cos y} + \frac{C(y)}{\partial(y)} = -\cancel{\ln\cos y}$

$C(y) = 0$

$f(1,0)-(e,\pi) = -\sin y \ln x \Big|^{1,0}$

$= -\sin 0 \cdot \cancel{\ln 1} - (-\sin\pi \cdot \ln e)$

$= \sin\pi \ln e = 0$

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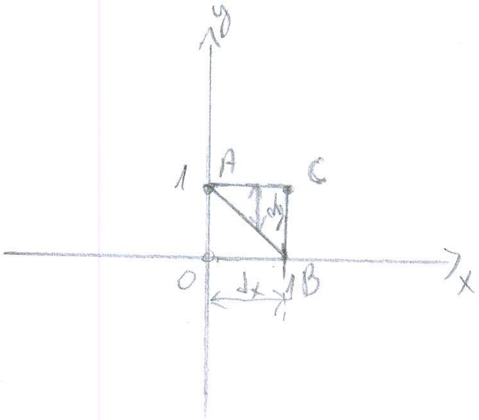
1. $\iint\limits_S e^{x+y}\,dx\,dy$ $\quad A(0,1) \quad B(1,0) \quad C(1,1)$



AB: $y-y_1=\dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1=\dfrac{0-1}{1-0}(x-0)$

$y-1=-x$

$y=-x+1$

0

$$\int_0^1\int_{-x+1}^1 e^{x+y}\,dx\,dy$$

$$\int_0^1\int_{-x+1}^1 e^x\cdot e^y\,dx\,dy = \int_0^1 e^y\,dy\int_{-x+1}^1 e^x\,dx = (e^y)\Big|_0^1\cdot(e^x)\Big|_{-x+1}^1 = (e^1-e^0)(e^1-e^{-x+1})$$

$$=(e-1)(e-e^{-x+1})$$

2. $x^2+y^2=4$ $\quad z=y \quad z=x-2$

$r^2\cos\varphi + r^2\sin\varphi = 4$

$r^2(\cos\varphi + \sin\varphi) = 4$

$r^2 = 4/r$

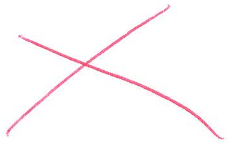
$r = \pm 2$

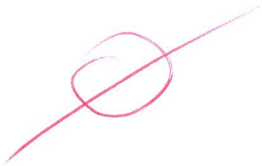
$r \in [0,2]$

$y = x-2$

$r\sin\varphi = r\cos\varphi - 2$

$r\sin\varphi - r\cos\varphi = -2$





MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

IME I PREZIME: BORIS DURBIĆ

BROJ INDEKSA: 57640

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. ~~20~~ 15

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$$\iint\limits_{\widehat{\partial C}} 2x \, dy dz$$

5. Izracunati $\displaystyle\int\limits_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} \, dx + \ln x \cos y \, dy$ 20

Ukupno: 15

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1}, \; n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left\| \frac{a+x}{a-x} \right\| + C$
$\int \frac{dx}{x} = \ln \|x\| + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left\| \frac{x-a}{x+a} \right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\| x + \sqrt{x^2 \pm a^2} \right\| + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln \|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln \|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax - x^2}} = \arccos \left(1 - \frac{x}{a}\right) + C$
$\int \tan x \, dx = -\ln \|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2} \, dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left(x + \sqrt{x^2 \pm a^2} \right) \right]$
$\int \cot x \, dx = \ln \|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2 - x^2} \, dx = \frac{1}{2} \left[x\sqrt{a^2 - x^2} + a^2 \arcsin \left(\frac{x}{a} \right) \right] + C$

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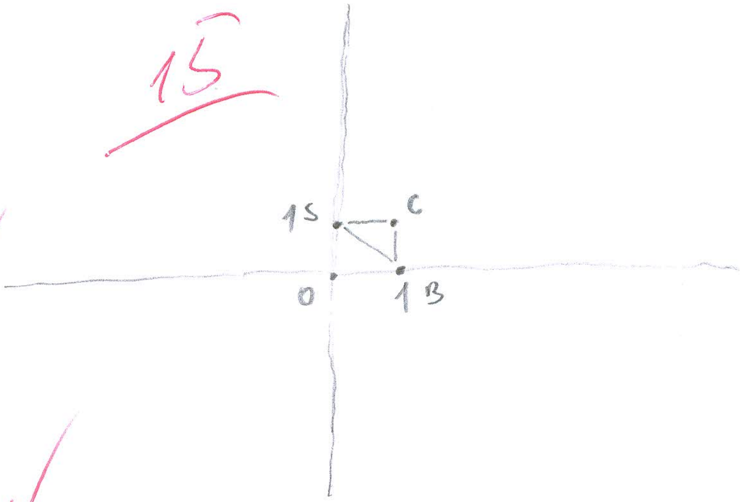
$f(t)$	$F(s) = \mathcal{L}[f](s)$	$f(t)$	$F(s) = \mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2 - a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2 - a^2}$
t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q) \, dq$
$t e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{F(s)}{s}$
$(1 - at) e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s) - f(0)$
$\sin(at)$	$\frac{a}{s^2 + a^2}$	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$
$\cos(at)$	$\frac{s}{s^2 + a^2}$	$f'''(t)$	$s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$

1. $\iint\limits_S e^{x+y}\,dx\,dy$ S(0,1) B(1,0) C(1,1)

15



$$\int_0^1\int_{-x+1}^1 e^{x+y}\,dx\,dy$$

$$\int_0^1\int_{-x+1}^1 e^x\cdot e^y\,dx\,dy$$

$$\int_0^1 e^x\cdot(e^1-e^{-x+1})\,dx$$

$$\int_0^1 e^x(e^1-e^{-x+1})\,dx$$

$$\int_0^1 e^{x+1}-\left(\frac{e^x}{e^x}\cdot e^1\cdot e^x\right)$$

$$\int_0^1 e^{x+1}-e^{1+x}\,dx$$

$$\int_0^1 dx = x = 1-0$$

$$= 1$$

$\overline{SB}$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}\cdot(x-x_1)$$

$$y-1=\frac{0-1}{1-0}(x-0)$$

$$y-1=-1\cdot x$$

$$y=-x+1$$

$\overline{SC}$

$$y-1=\frac{1-1}{1-0}(x-1)$$

$$y=1$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: VICE VIŠIĆ **BROJ INDEKSA:** 57102

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y}\mathrm{d}x\mathrm{d}y$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20 15
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20 0
$$x'''(t)+x'(t)=0,\quad x(0)=x''(0)=1,\ x'(0)=0.$$
4. Neka je C cilindar zadan sa $C=\{(x,y,z):(x+2)^2+(y-3)^2\le 1,\ -1\le z\le 1\}$. Izračunati plošni integral 20
$$\iint_{\partial C} 2x\,\mathrm{d}y\mathrm{d}z$$
5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,\mathrm{d}x+\ln x\cos y\,\mathrm{d}y$ 20

Ukupno: 15

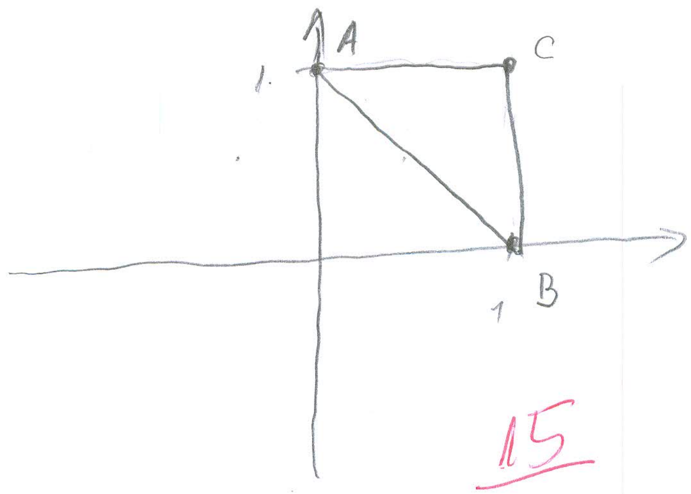
Tablica integrala

$\int \mathrm{d}x = x + C$	$\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x + C$	$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1},\ n\ne -1$	$\int \frac{\mathrm{d}x}{\sin^2 x} = -\cot x + C$	$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{\mathrm{d}x}{x} = \ln\|x\| + C$	$\int \sinh x\,\mathrm{d}x = \cosh x + C$	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x\,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$\int \cosh x\,\mathrm{d}x = \sinh x + C$	$\int \frac{\mathrm{d}x}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\,\mathrm{d}x = -\cos x + C$	$\int \tanh x\,\mathrm{d}x = \ln\|\cosh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\,\mathrm{d}x = \sin x + C$	$\int \coth x\,\mathrm{d}x = \ln\|\sinh x\|$	$\int \frac{\mathrm{d}x}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\,\mathrm{d}x = -\ln\|\cos x\|$	$\int \frac{\mathrm{d}x}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,\mathrm{d}x = \ln\|\sin x\|$	$\int \frac{\mathrm{d}x}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,\mathrm{d}q$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,\mathrm{d}\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1. Trokut $A(0,1)$ $B(1,0)$ $C(1,1)$ $\iint\limits_S e^{x+y}\,dx\,dy$



$\vec{AB}$ ... $y-y_1 = \dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$

$y-1 = \dfrac{0-1}{1-0}(x-0)$

$y-1 = -x$

$y = -x+1$

15

$$\int_0^1 \int_{-x+1}^1 e^{x+y}\,dy\,dx = \int_0^1 e^x\,dx \cdot \int_{-x+1}^1 e^y\,dy = e\cdot e - e^{-x+1} = e^2 - e^{-x+1}$$

---

3. $x'''(t) + x'(t) = 0$ $\quad x(0) = x''(0) = 1$ $\quad x'(0) = 0$

$= s^3 X(s) - s^2 \underset{1}{x(0)} - s\underset{0}{x'(0)} - \underset{1}{x''(0)} + sX(s) - \underset{1}{x(0)} = 0$

$= s^3 X(s) - s^2 - 1 + sX(s) - 1 = 0$

$= s^3 X(s) + sX(s) - s^2 - 2 = 0$

$X(s)(s^3+s) = s^2+2$

$X(s)\, s^2(s+1) = s^2+2 \quad / \cdot \dfrac{1}{s^2(s+1)}$

$X(s) = \dfrac{s^2+2}{s^2(s+1)}$

$s^2+2 = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{(s+1)} \quad / s^2(s+1)$

$s^2+2 = A(s+1) + Bs(s+1) + Cs^2$

$s^2+2 = As + A + Bs^2 + Bs + Cs^2$

$(B+C)s^2 + (A+B)s + A$

$B+C=1 \qquad -2+C=1 \qquad \boxed{C=3}$

$A+B=0 \qquad 2+B=0$

$\boxed{A=2} \qquad \boxed{B=-2}$

$= \dfrac{1}{s^2}\cdot A + \dfrac{1}{s}\cdot B + \dfrac{1}{s+1}\cdot C$

$= 2t - 2\cdot 1 + 3e^{-t}$

$= 2t + 3e^{-t} - 2$

2. Volumen $x^2+y^2=4$ $z=y$ $z=x-2$

$x^2+y^2=4$

$x^2+y^2=r^2$

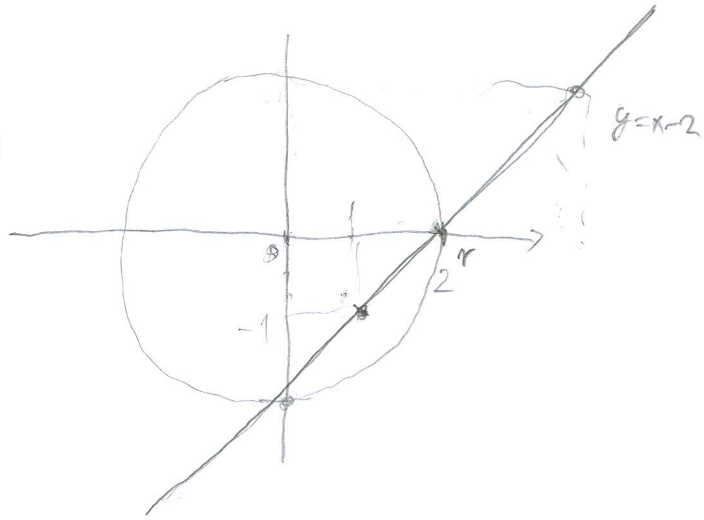
$r^2=4$

$r=2$

$z=y$

$y=x-2$

x	0	1	2	4
$y=x-2$	-2	-1	0	2



Cilindrične koordinate

$x = r\sin\varphi$

$y = r\cos\varphi$

$z = z$

$dxdydz = r\,dr\,d\varphi\,dz$

$$V = \int_0^2 \int_0^{2\pi} \int_{x-2}^{0} r\,dr\,d\varphi\,dz = \int_0^2 \int_0^{2\pi} r^{x-2}\,d\varphi\,dz =$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: NINO MIKULANDRA **BROJ INDEKSA:** 57645

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: 20
$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=1,\ x'(0)=0.$$
4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2\le 1,\ -1\le z\le 1\}$. Izračunati plošni integral 20
$$\iint_{\widehat{\partial C}} 2x\, dy dz$$
5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\, dx + \ln x \cos y\, dy$ 20

Ukupno:

Tablica integrala

$\int dx = x+C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\| + C$
$\int \frac{dx}{x} = \ln\|x\| + C$	$\int \sinh x\, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x\, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\|x+\sqrt{x^2\pm a^2}\right\| + C$
$\int \sin x\, dx = -\cos x + C$	$\int \tanh x\, dx = \ln\|\cosh x\|$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
$\int \cos x\, dx = \sin x + C$	$\int \coth x\, dx = \ln\|\sinh x\|$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x\, dx = -\ln\|\cos x\|$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\, dx = \ln\|\sin x\|$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2\arcsin\left(\frac{x}{a}\right)\right] + C$

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e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\, dq$
$t\, e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\, d\tau$	$\frac{F(s)}{s}$
$(1-at)\, e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
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$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

1.) $\iint\limits_S e^{x+y}\,dx\,dy$

$A(0,1),\ B(1,0),\ C(1,1)$

$$= \int_0^1\int_{1+2x}^1 e^{x+y}\,dx\,dy = \int_0^1 \left[\frac{e^{x+y}}{x}\right]_{y=1+2x}^{y=1} dx =$$

$$= \int_0^1 \frac{e^x}{x} - \frac{e^{x(1+2x)}}{x}\,dx = \int_0^1 \frac{e^x}{x}\,dx - \int_0^1 \frac{e^{x(1+2x)}}{x}\,dx$$

MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

IME I PREZIME: MARKO BUBIČIĆ **BROJ INDEKSA:** 54768-2007

Grupa xxoox

POPUNJAVA NASTAVNIK

Broj ↓ bodova

1. Izračunati dvostruki integral $\iint\limits_S e^{x+y} dx dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. 20
2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. 20
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4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2 \le 1, -1 \le z \le 1\}$. Izračunati plošni integral 20
$$\iint\limits_{\widehat{\partial C}} 2x \, dy dz$$
5. Izracunati $\displaystyle\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x} dx + \ln x \cos y \, dy$ 20

Ukupno: 0

Tablica integrala

$\int dx = x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1},\ n \neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left\lvert \frac{a+x}{a-x} \right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x \, dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \cosh x \, dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left\lvert x + \sqrt{x^2 \pm a^2} \right\rvert + C$
$\int \sin x \, dx = -\cos x + C$	$\int \tanh x \, dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
$\int \cos x \, dx = \sin x + C$	$\int \coth x \, dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right) + C$
$\int \tan x \, dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2}\left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln\left(x + \sqrt{x^2 \pm a^2}\right)\right]$
$\int \cot x \, dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left[x\sqrt{a^2-x^2} + a^2 \arcsin\left(\frac{x}{a}\right)\right] + C$

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t	$\frac{1}{s^2}$	$e^{-at} f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(a t)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\, dq$
$t\, e^{-at}$	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\, d\tau$	$\frac{F(s)}{s}$
$(1-at)\, e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

$y^2 = 4$ $\qquad z = y$

$\sqrt{4}$ $\qquad r \in [0, 2]$

2 $\qquad \varphi \in [0, 2\pi]$

$\qquad z \in [y, x-2]$

$\cos\varphi$

$\sin\varphi$

$$\int_0^{2\pi} d\varphi \int_0^2 r\,dr \int_{r\sin\varphi}^{r\cos\varphi - 2} dz = \int_0^{2\pi} d\varphi \int_0^2 (r\cos\varphi - 2 - (r\sin\varphi))\, r\,dr$$

$$= \int_0^{2\pi} (r(\cos\varphi - \sin\varphi) - 2) \frac{r^2}{2} \Big|_0^2 d\varphi$$

$$= \frac{1}{2}\int_0^{2\pi} (r^3(\cos\varphi - \sin\varphi - 2r^2)) \Big|_0^2 d\varphi$$

$$= \frac{1}{2}\int_0^{2\pi} (8(\cos\varphi - \sin\varphi - 8))\, d\varphi$$

$$= \frac{1}{2}\int_0^{2\pi} (8\cos\varphi - 8\sin\varphi - 64)\, d\varphi$$

$$= \frac{1}{2}\left(8\cos\varphi - 8\sin\varphi - 64\right)\Big|_0^{2\pi}$$

$$= \frac{1}{2}\left(8\cos 2\pi - 8\sin 2\pi - 64 - (8\cos 0 - 8\sin 0 - 64)\right)$$

$$= \frac{1}{2}(8 - 64 - 1 - 64)$$

$$= \frac{1}{2}(-121) = -\frac{121}{2}$$

NEGATIVAN VOLUMEN

$\emptyset$

3. $x'''(t) + x'(t) = 0 \qquad x(0) = x''(0) = 1 \qquad x'(0) = 0$

$$s^3 X(s) - s^2 x(0) - s x'(0) - x''(0) + (s X(s) - x(0)) = 0$$

$$s^3 X(s) - s^2 - 1 + s X(s) - 1 = 0$$

$$X(s)(s^3 + s) = s^2 \pm 2$$

$$X(s)(s(s^2+1)) = s^2 - 2 \quad / : (s(s^2+1))$$

$$X(s) = \frac{s^2 - 2}{s(s^2+1)}$$

$$\frac{s^2 - 2}{s(s^2+1)} = \frac{A}{s} + \frac{Bs + C}{s^2+1}$$

$$s^2 - 2 = As^2 + A + Bs^2 + Cs$$

$1 = A + B \qquad 1 = -2 + B$

$-2 = A \qquad 1 + 2 = B$

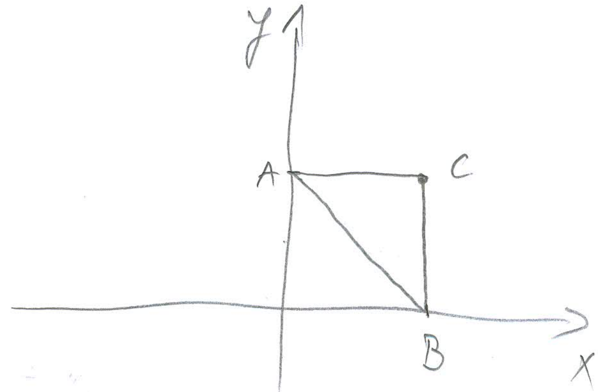
$0 = C \qquad B = 3$

$$X(s) = -2\frac{1}{s} + 3\frac{s}{s^2+1}$$

$$\underline{x(t) = -2 + 3\cos t}$$

① $\iint\limits_S e^{x+y}\,dx\,dy$

A(0,1)
B(1,0)
C(1,1)



$\int_0^1 e^{x+y}\,dy + \int_{1-y}^1 e^{x+y}\,dx =$

$\int_0^1 e^{x+y}\left(\frac{e^x}{\ln e}\right)\Big|_{1-y}^{1} dy =$

$= \int_0^1 e^{x+y}\left(\frac{e}{\ln e} - \frac{e^{1-y}}{\ln e}\right) dy$

$= \int_0^1 e^{x+y}\left(e - e^{1-y}\right) dy$

$= \frac{e^y}{\ln e}\left(e - e^{1-y}\right)\Big|_0^1$

$= e - 1\left(e - e^{1-1}\right) - \left(e - e^1\right)$

$= e^2 - e - e - 1 - \cancel{e} + \cancel{e}$

$= e^2 - 2e - 1$

AC: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 1 = \frac{1-1}{1-0}(x - 0)$

$y - 1 = \frac{0}{1}(x - 0)$

$y - 1 = 0$

$y = 1$

BC: $x = 1$

AB: $y - 1 = \frac{0-1}{1-0}(x - 0)$

$y - 1 = \frac{-1}{1}(x - 0)$

$y - 1 = -x$

$y = -x + 1$

$x = 1 - y$

5. $$\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x \cos y\,dy$$

$$w = \begin{bmatrix} \frac{\sin y}{x} \\ \ln x \cos y \end{bmatrix} = -\operatorname{grad} f$$

$$f(x) = -\frac{\sin y}{x} \quad \int dx$$

$$f = -\frac{\sin y}{1} \cdot x^{-1}$$

$$f =$$

$$dy\left[0 + c(y)\right] = -\ln x \cos y \quad \int dy$$

$$c(y) = -\sin y$$

$$f = \; ?$$

④ $(x+2)^2+(y-3)^2 \leq 1 \qquad -1 \leq z \leq 1$

$R^2 \leq 1$

$\iint 2x \, dy \, dz = ? \qquad z \in [-1, 1]$

$d\varphi \, [0, 2\pi]$

$2\left(\int_0^{2\pi} x \, d\varphi + \int_{-1}^{1} x \, dz\right) = \qquad r \in [0, 1]$

$2\left(\int_0^{2\pi} x \left(\frac{x^2}{2}\right)\Big|_{-1}^{1} d\varphi\right) = 2\left(\int_0^{2\pi} x \, d\varphi\right) = 2\left(\frac{x^2}{2}\Big|_0^{2\pi}\right)$

$= 2\left(\frac{4\pi^2}{2}\right) = \frac{8\pi^2}{2}$