MATEMATIKA 3: Ispit se održava sukladno objavljenim pravilima. Na snazi je Pravilnik o stegovnoj odgovornosti studenata. **PIŠITE DVOSTRANO!**

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Grupa xxoox
POPUNJAVA NASTAVNIK
Broj ↓ bodova

1. Izračunati dvostruki integral $\iint_S e^{x+y}\,dx\,dy$, gdje je S trokut s vrhovima $A(0,1)$, $B(1,0)$, $C(1,1)$. — 20 ~~20~~ 15

2. Izracunati volumen tijela omeđenog valjkom $x^2+y^2=4$ i ravninama $z=y$ i $z=x-2$. — 20

3. Koristeći Laplaceovu transformaciju riješiti diferencijalnu jednadžbu: — 20

$$x'''(t)+x'(t)=0, \quad x(0)=x''(0)=1,\ x'(0)=0.$$

4. Neka je C cilindar zadan sa $C=\{(x,y,z) : (x+2)^2+(y-3)^2\le 1,\ -1\le z\le 1\}$. Izračunati plošni integral — 20

$$\iint_{\partial\overset{\frown}{C}} 2x\,dy\,dz$$

5. Izracunati $\int_{(1,0)}^{(e,\pi)} \frac{\sin y}{x}\,dx + \ln x\cos y\,dy$ — ~~20~~

Ukupno: ~~55~~

Tablica integrala

$\int dx = x+C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a}+C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1},\ n\neq -1$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$	$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert + C$
$\int \frac{dx}{x} = \ln\lvert x\rvert + C$	$\int \sinh x\,dx = \cosh x + C$	$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert + C$
$\int a^x\,dx = \frac{a^x}{\ln a}+C$	$\int \cosh x\,dx = \sinh x + C$	$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left\lvert x+\sqrt{x^2\pm a^2}\right\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \tanh x\,dx = \ln\lvert\cosh x\rvert$	$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a}+C$
$\int \cos x\,dx = \sin x + C$	$\int \coth x\,dx = \ln\lvert\sinh x\rvert$	$\int \frac{dx}{\sqrt{2ax-x^2}} = \arccos\left(1-\frac{x}{a}\right)+C$
$\int \tan x\,dx = -\ln\lvert\cos x\rvert$	$\int \frac{dx}{\cosh^2 x} = \tanh x + C$	$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}\left[x\sqrt{x^2\pm a^2} \pm a^2\ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$
$\int \cot x\,dx = \ln\lvert\sin x\rvert$	$\int \frac{dx}{\sinh^2 x} = -\coth x + C$	$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2-x^2}+a^2\arcsin\left(\frac{x}{a}\right)\right]+C$

Tablica Laplaceovih transformacija:

$f(t)$	$F(s)=\mathcal{L}[f](s)$	$f(t)$	$F(s)=\mathcal{L}[f](s)$
1	$\frac{1}{s}$	$\sinh(at)$	$\frac{a}{s^2-a^2}$
c	$\frac{c}{s}$	$\cosh(at)$	$\frac{s}{s^2-a^2}$
t	$\frac{1}{s^2}$	$e^{-at}f(t)$	$F(s+a)$
t^n	$\frac{n!}{s^{n+1}}$	$f(at)$	$\frac{1}{a}F(\frac{s}{a})$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$t^n f(t)$	$(-1)^n F^{(n)}(s)$
e^{-at}	$\frac{1}{s+a}$	$\frac{f(t)}{t}$	$\int_s^\infty F(q)\,dq$
te^{-at}	$\frac{1}{(s+a)^2}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
$(1-at)e^{-at}$	$\frac{s}{(s+a)^2}$	$f'(t)$	$sF(s)-f(0)$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$f'''(t)$	$s^3F(s)-s^2f(0)-sf'(0)-f''(0)$

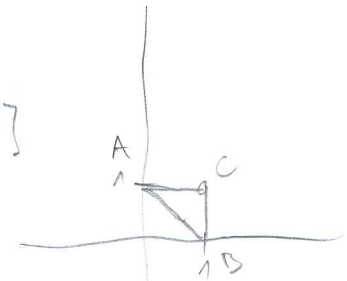
1.

$$\iint_S e^{x+y}\,dx\,dy$$

$A(0,1)$ $B(1,0)$ $C(1,1)$

$X \in [0,1]$

$Y \in [1-x, 1]$



$$\int_0^1 \int_{1-x}^{1} e^{x+y}\,dy\,dx = \int_0^1 e^x \cdot \left(e^1 - e^{1-x}\right) dx$$

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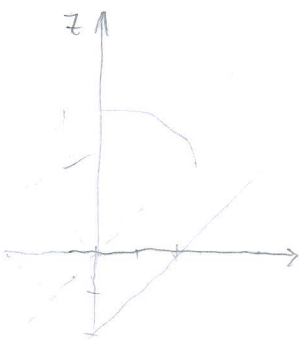
2.

$x^2+y^2=4$; $z=y$, $z=x-2$

$r \in [0,2]$

$\varphi \in [0, 2\pi]$

$z \in [r\cos\varphi - 2,\ r\cos\varphi]$



$$\int_0^{2\pi}\int_0^{2}\int_{r\cos\varphi-2}^{r\cos\varphi} r\,dz\,dr\,d\varphi$$

$$\int_0^{2\pi}\int_0^{2} r^2\cos\varphi - r^2\cos\varphi + 2r\,dr\,d\varphi = \int_0^{2\pi}\left(\frac{8}{3}\cos\varphi - \frac{8}{3}\cos\varphi + 4\right)d\varphi = \frac{8}{3}\sin(2\pi) - \frac{8}{3}\sin(0) + 8\pi$$

$= 8\pi = 25{,}13$

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